Thoe

An experimental analysis of environmental
effects on pressure sensing systems for
oscillating pressures.

AN EXPERIMENTAL ANALYSIS OF ENVIRONMENTAL EFFECTS

ON PRESSURE SENSING SYSTEMS FOR OSCILLATING PRESSURES

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SUMMARY

Experimental tests were conducted using a generated square
pressure wave to determine the environmental effects on pressure
sensing systems for oscillating pressures. Square wave pressure
oscillations from approximately 25 to 120 cycles per second and
about six inches of mercury in amplitude were used for various
configurations of the tube connecting the pressure sensing instru-
ment to the source of pressure oscillation.

It was necessary to resort to harmonic analysis of the
pressure waves obtained on oscillograms to determine the three basic
results of attenuation, phase lag, and the degree and kind of
distortion caused by the connecting tube system. Insufficient
time was available to analyze a sufficient number of oscillograms
which would compile enough information to enable the designer to
select the proper configuration which would give a satisfactory
response over a desired frequency range or to eliminate undesired
resonances when only a mean pressure level is desired.

It was concluded that any arbitrarily produced pressure
wave can be used to determine attenuation, phase lag, and distortion
through use of harmonic analysis and comparison with the wave
obtained with the pressure sensing instrument connected directly
to the pressure wave source. The attenuation and phase lag for
the first six harmonics at a frequency of 30 CPS for four finite

tube lengths were determined experimentally and those for the
fundamental were shown to be in fair agreement with that obtained
theoretically as obtained from the graphs prepared by A. S. Iberall
in reference (4).

This investigation was conducted in the Mechanical
Engineering Instrument Laboratory of the University of Minnesota
by R. R. Thoe under the guidance of the Mechanical Engineering
Department Staff during the Spring of 1952.

AN EXPERIMENTAL ANALYSIS OF ENVIRONMENTAL EFFECTS ON
PRESSURE SENSING SYSTEMS FOR OSCILLATING PRESSURES

INTRODUCTION

The measurement of fluctuating fluid pressures presents
a problem in many present-day applications. Numerous devices have
been utilized but the demand for more accurate methods for relatively
larger amplitudes and higher frequencies of pressure oscillations
have created many new applications of principles for pressure
sensing systems for specific problems.

In measuring rapidly changing pressures, one of the more
important considerations is the dynamic characteristics of the elastic
system of the pressure sensing device, in particular the natural
frequency and the amount of damping present. The displacement of
the pressure sensing system should be in proportion to and in phase
with the applied pressure with no relative distortion in amplitude
or phase of the harmonic component of the oscillating pressure
source up to the highest component it is desired to measure.

There are many other important considerations which include:

(1) Small, portable, and inexpensive as possible.

(2) Sense gauge or differential pressures.

(3) Have constant calibration over period of time.

(4) Simplicity in operation.

(5) High sensitivity.

(6) Ample range in amplitude and frequency.

(7) Adds negligible volume of gas to system being
     measured.

(8) Contains negligible hysterisis.

(9) Unaffected by thermal, electrical, magnetic, and
     mechanical interference.

(10) Ability to utilize long cables if necessary.

(11) Direct and permanent method of presenting recorded
     pressures.

No known device meeting all the above requirements exists
at present, but rather there are many devices which are more or less
satisfactory for the field for which designed. Some of the pressure
sensing systems which have found considerable use are briefly dis-
cussed in Ref. (1) and (2).

Often it is impossible to install the pressure sensing
element directly at the point of measurement. It then becomes
necessary to connect the pressure sensing element to the source by
a connecting tube. This connecting tube introduces the possibility
of errors due to resonance or attenuation in the tube. This results
from the fact that the air column in the tube has a definite mass
inertia, elasticity, and can dissipate energy with its own motion
such that wave motion can be propagated along its tube length.

Several theoretical and experimental investigations have been undertaken in an effort to determine the effect of connecting tube on the pressure response indicated by the sensing instrument.

Israel Taback, Ref. (3), compared the pressure response obtained theoretically with that obtained experimentally for a steady state sinusoidal oscillating pressure with various environmental changes including the following all at varying frequencies from zero to 70 cycles per second for several tube lengths:

    (1) Simple tube system of negligible instrument volume.

    (2) Tube with inlet restriction.

    (3) Appreciable instrument volume.

Theoretical results were obtained using the electrical alternating current R-L-C circuit analogy. Unfortunately this method requires a considerable amount of tedious computation for even the simplest systems and insufficient information exists to warrant the validity of these solutions for large amplitude oscillations.

Experimental results compared favorably with that computed for various tube lengths, diameters, and appreciable instrument volume at frequencies from zero to 70 cycles per second. Experimental results showed that an inlet restriction had a damping effect at resonance but negligible effect at antiresonance. The effect of an appreciable instrument volume was to lower the resonant frequency

of the system and decreased the amplitude of the recorded pressures
through most of the frequency range investigated.

In this report it was concluded

that for accurate dynamic pressure measurement the
first resonant frequency of the pressure measuring
system should be kept well above the highest pressure
frequency to be measured.

Since the resonant frequency is inversely proportional to the
length of the connecting tube, the simplest way to accomplish this
is to place the pressure sensing instrument as close as possible to
the source.

A theoretical investigation of the attenuation and lag
of an oscillatory pressure variation for a finite connecting tube
length to the pressure sensing instrument was made by A. S. Iberall,
Ref. (4). A sinusoidal pressure oscillation was assumed at the
source which was connected to the pressure sensing device by a
connecting tube. The problem was attacked by first assuming an
incompressible viscous fluid such that Poiseuilles law of viscous
resistance was valid throughout the system. These results were
then corrected for compressibility, finite-pressure amplitudes,
appreciable acceleration, and effects for finite length of tube,
and heat transfer through the tube. The correction for
compressibility introduced a time constant and attenuation factor
dependent on the tube and instrument volumes. Finite pressure
excess amplitudes about the mean pressure introduced harmonic
distortion proportional to the amplitude of the pressure excess

in the case of compressible fluids, but the attenuation of the fundamental appeared to be essentially independent of the amplitude of the oscillation. The inertia of the fluid through its influence on fluid acceleration modified the time constants of the system affecting both the attenuation of the fundamental and the amount of harmonic distortion. It was shown that if the fluid inertia is large an undamped system is present permitting elementary acoustic theory to be used, but when fluid inertia was negligible the transmission tube became a highly damped system. In short tubes a correction must be introduced to allow for the end effects resulting from fluid acceleration at the ends of the tube which further distort the wave form. At low frequencies, an isothermal process was assumed, but at higher frequencies where heat transfer cannot be assumed perfect and the process can still not be assumed to be adiabatic, a further correction for this polytropic process was introduced.

The results were presented in a series of graphs which permit the designer to determine certain dimensions for a pressure sensing system to be used with a given sinusoidal oscillating pressure to obtain a desired amplitude and lack of distortion, or to determine the attenuation and lag of a given pressure sensing system.

This theory could be applied to other types of pressure oscillations such as a square wave pressure source by resorting to

Fourier analysis, although it would require a great amount of tedious and arduous mathematical computation.

NACA TN 1988, Ref. (5), develops theoretically and compares experimental results of the transient behavior of lumped-constant system for sensing transient pressures. The pressure sensing system consisted of a tube connected to a reservoir where the pressure sensing instrument was located. A short straight tube whose volume was much smaller than the reservoir volume was used so that the dead time, length of tube divided by the speed of sound, was negligible and the volume flow through the system depended essentially only on the compressibility of the gas in the reservoir. Thus it was assumed that space variables had no effect and that the pressure disturbances occurred instantaneously through-out the system with varying magnitudes. In addition for the theoretical analysis, the process was assumed to be adiabatic and to consist of only small pressure changes.

Experimental data was obtained by interrupting an air stream directed at the open end of the tube. This was accomplished by using a revolving slotted disk between the stream source and the open end of the tube. The pressure response in the reservoir was sensed by a commercial device which presented its readings on an oscilloscope.

One of the primary purposes of this investigation was to establish a method for determining design parameters for a control

loop system encountering transient pressures. From the results obtained it was shown that the differential equations utilized in the theoretical analysis were sufficiently valid for design purposes for similar conditions in practice.

For a given gas under specified conditions, the undamped natural frequency and damping ratio are functions of the tube length, tube radius, and reservoir volume. Thus for given desired values of frequency and damping ratio, when one of these system dimensions is limited by practical reasons, the other two dimensions are uniquely determined. If two of the dimensions are restricted, the third can be determined for either frequency or damping ratio. If tube length and reservoir volume have already been reduced to the practical minimums, the undamped natural frequency can be increased only by increasing the tube radius. Unfortunately this may decrease the damping ratio too much (for control loops). However, the damping ratio was increased without changing the system dimensions or natural frequency through the addition of an increased resistance in the tube such as a wire mesh or orifice.

An experimental investigation was conducted, Ref. (6), using a square-wave pressure valve to determine experimentally some of the effects of changes in important design parameters on a balanced-pressure diaphragm indicator for measuring transient pressure changes. The square wave pressure generator consisted essentially of a rotating slotted disk which connected the sensing

reservoir alternately to two reservoirs at two different known
static pressure levels. Limitations of the equipment used prevented
findings of much value to the immediate field of this study. The
clamped diaphragms introduced a zero shift error due to elastic
stress which could be eliminated by the use of free unclamped
diaphragms. However, the lower natural frequency of free diaphragms
limited the frequency of pressure oscillations to a rather low value.
Considerable erratic behavior was detected in the response of
diaphragms to a change in pressure in time, sometimes lagging much
more than other times. The cause of this erratic response time
was not determined and may have been due to the square-wave generator
or pick-up system, although varying the diaphragm mass, motion, and
damping had no apparent effect on the degree of erratic response
time.

The present investigation is an experimental analysis of
environmental effects on pressure sensing systems for oscillating
pressures. It was conducted through use of a square wave pressure
generator designed and previously used by E. M. Smith at the
University of Minnesota and is described in the Test Equipment
section of this report. This generator was driven by a variable
speed motor so that the fundamental wave frequency could be
controlled at will.

Three different types of pressure sensing devices were
originally contemplated to enable a comparison and check of the

data obtained. The three pressure pick-up systems selected were:

(1) The Statham Gage.

(2) The Electro-Pressuregraph.

(3) The Mechano-Electronic Transducer.

These three systems were selected largely for their high natural frequency, thus permitting investigation of pressure oscillations to a relatively high frequency. In addition, all three systems permit pressure readings to be easily presented on the cathode ray tube oscilloscope. However, due to technical difficulties and shortage of time, the mechano-electronic transducer was not used in this investigation.

It was the intention of the author to vary those environmental configurations which occur frequently in pressure recording installations in actual practice. The primary changes investigated were changes in the length of the tube connecting the instrument to the wave generator at frequencies from 25 to 120 cycles per second. Tube lengths tested were standard 1/4 inch outer diameter copper tubing and were of 12, 24, 42, and 60 inches in length. Zero tube length test runs were made to which the other runs were compared. In addition test runs were made of larger diameter tubes (3/8 and 1/2 inch O.D.), but due to the fittings on the instrument and the square wave generator, these larger diameter tubes had to be restricted at their ends down to the same diameter size used for the 1/4 inch tubing. Several tests were conducted on the efficiency of various plenum chamber configurations to determine effectiveness in damping of oscillations.

The arbitrary character of the initial pressure wave complicated the analysis of the oscillograms obtained. Since the generated wave was not a perfect square wave, comparison of a wave obtained from any given configuration to that of the zero tube length wave for the same frequency required harmonic analysis to obtain any reliable information. This was a task requiring more time than available so that this report was narrowed down eventually to the case of one frequency, 30 CPS, for tube lengths of zero, 12, 24, 42, and 60 inches in the determination of attenuation, phase lag, and distortion caused by finite tube lengths.

This investigation was conducted by R. R. Thoe at the Internal Combustion Engines Instrument Laboratory of the University of Minnesota, during the Spring of 1952.

TEST EQUIPMENT

The experimental tests were conducted using a square
pressure wave generator driven by a variable speed motor. Through
this both the frequency and amplitude of the square pressure wave
could be varied at will. A pressure transducer interpreted the
wave and presented it on a cathode ray tube oscilloscope, Dumont
Type 304-H, from which oscillograms were made through utilization
of a Dumont oscillogram camera. Figs. 1, 2, and 3 show the general
set-up of the test equipment.

Three different pressure sensing devices were contemplated
for this investigation: the Statham gage, the Electro-Pressuregraph,
and the mechano-electronic transducer. Tests were made using the
first two instruments, but the mechano-electronic transducer was
not used due both to lack of time and difficulties in obtaining the
tube and fabricating a satisfactory linkage system in the pressure
pickup for the tests contemplated.

The Statham gage, model P6-4D, pictured in Figs. 1 and 2,
utilizes an electrical strain gage mounted on a pressure diaphragm.
This strain gage is one element of an electrical bridge circuit
whose output is fed directly into the oscilloscope where full
D. C. amplification was used throughout all test runs using the
Statham gage.

The Electro-Pressuregraph, manufactured by the Electro-Products Laboratories of Chicago, Illinois, is of the capacitor type in which the pressure diaphragm forms the variable capacitance element and is pictured in Fig. 3. For the pressure range investigated a diaphragm thickness of 0.010 inches was used.

All tests were conducted using the square pressure wave generator sketched in Fig. 4. This consisted of a rotating valve element mounted in a housing with ports drilled every 90 degrees along the circumference of the housing. The rotating element had one hole drilled diametrically and one axially connecting the diametrically drilled hole with the end hole, 5, in the housing. As the rotating element revolves, the diametrical hole connects the port at 5 first to the tank pressure, p_g, applied at 1 and 3, causing the pressure at 5 to increase to pressure, p_g, until the diametrical hole is uncovered by ports 2 and 4 exhausting the entrapped pressure to atmospheric pressure, p_0. Thus it can be seen that a square pressure wave is generated with a frequency twice that of the frequency of rotation of the rotating element.

A stiff hose coupling was used first between the shaft of the motor and the rotating element of the wave generator to eliminate vibration from the motor during the Statham gage test runs. It was believed that a considerable amount of variable lag between the generator and the motor shaft on which the timing circuit was mounted resulted from such a flexible coupling.

Therefore, a universal joint coupling was substituted which had a relatively constant lag with revolution speed and was used throughout the Electro-Pressuregraph test runs.

The gage pressure, p_g, was supplied by a cylinder of CO_2 gas expanded through two valves to facilitate close control of the gage pressure. Fig. 5 is a schematic sketch of the gage pressure system. By opening valve A, pressure was admitted to gage (1), and by carefully opening valve B any desired gage pressure as read on the U-tube manometer, M, could be placed throughout the manifold system. Valve C permits exhaust to the atmosphere and was used during static calibration of the instruments. Valve D opened the pressure to the wave generator and was open during the dynamic tests. Fitting S_c was the location of the instrument during calibration runs, the instrument being replaced by a plug during dynamic test runs.

The Statham gage, being a resistance bridge circuit instrument, lends itself to a convenient method of calibration. Fig. 6 is a schematic sketch of the electrical circuits used. The resistance R_c across an input and output terminal of the bridge serves as a means of indicating an arbitrary calibration pressure line on each oscillogram. When the switch, S, is closed the oscilloscope trace jumps vertically to a value of a corresponding pressure. A resistance of $R_c = 270,000$ ohms was found to give a pressure indication of 6 inches of mercury gage. This pressure

line was placed on each test run picture along with the zero
pressure line to give a scale factor to each picture of the
Statham test runs.

To determine phase lag, an electric brush timer was
designed and installed on the motor shaft. Fig. 6 illustrates
the principle of the brush timer. Since there are two cycles
of the wave for each revolution of the shaft a signal every
180 degrees of rotation of the shaft was desirable. A brass
collar, with a 180 degree slice down one side filled with plastic
was mounted on the shaft. The two brushes complete the timer
circuit when both brushes are in contact with the brass and open
the circuit when the one brush contacts the plastic. The
remainder of the timer electrical circuit consisted of a battery
and a variable resistance load and a step-up transformer. The
secondary of the transformer fed into the Z axis of the
oscilloscope and the common ground.

As the circuit is closed by the brushes a flow of
current goes through the primary of the transformer and during
the transient current flow, a voltage is generated in the
secondary transmitting a pulse to the Z axes of the oscilloscope
increasing the intensity of the trace on the oscilloscope screen
momentarily. When the brush contacts the plastic as the shaft
revolves, the circuit is opened and there is a sudden stoppage
of current in the primary. This transient again causes a

voltage to be generated in the secondary but in the opposite direction so that the trace on the oscilloscope screen is decreased in intensity. Thus a time signal was imposed on the oscilloscope screen to indicate when the wave cycle actually started at the generator.

The shielding and grounding of all electrical input cables to the oscilloscope was found to be essential to prevent the pick-up of extraneous signals on the oscilloscope screen.

TEST PROCEDURE

A minimum warm-up period of one hour for the oscilloscope,
strobotac, and constant voltage transformer, and of twenty minutes
for the Statham gage and Electro-Pressuregraph was observed before
all test runs to ascertain temperature equilibrium throughout the
system. This was found necessary due to the inherent tendency for
vertical drift in the Dumont 304-H oscilloscope. However, using
these precautions of sufficient warm-up time and the use of a
constant voltage transformer between the line voltage and the
oscilloscope, the vertical drift of the oscilloscope trace was
negligible and caused little trouble. Nevertheless, it was found
impossible to use this type oscilloscope without the constant
line voltage since vertical drifts as large as half an inch were
sometimes noticed in the matter of only two or three seconds when
used with a regular voltage supply source.

Two calibration runs were made for each series of test
runs as taken on each roll of film. One calibration was taken
at the start of the runs and another at the end of the film to
make sure that no change in calibration of the instrument occurred
during the test runs.

To calibrate the Statham gage, valve C and D of Fig. 5
were both closed and the instrument attached at S_c. A pressure
was imposed on the gage by opening valve B until a given pressure
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in the system. A picture was then taken of the deflection on the oscilloscope and the pressure recorded. Valve C was then opened to exhaust the pressure to the atmosphere and the trace on the oscilloscope was checked to ascertain that it had returned to its zero reference line. This was facilitated by placing a small mark on the oscilloscope screen which was easily discernible through the peephole of the recording camera hood when the trace was tangent to this mark. Other pressures were applied in a similar manner from zero gage pressure to a gage pressure of four pounds per square inch, the maximum allowable for this gage. All calibration lines were exposed on the same film frame through multiple exposures. It was found impractical to calibrate the instrument when connected to the wave generator as there was considerable leakage of pressure through the wave generator. This, of course, made it impossible to maintain a constant static pressure when the generator was not in operation.

The Electro-Pressuregraph was calibrated in the same manner with the exception that no device similar to the calibrated resistance, R_c, used for the Statham gage, was utilized due to the increased complexity of a capacitor circuit.

After the initial calibration was completed the gage was removed from the calibration position, S_c, of Fig. 5, which was then plugged, and the gage was attached to the output of the square wave generator. Oscillograms were made of fundamental

wave frequencies from approximately 25 to 100 cycles per second
in steps of approximately five cycles per second.

The configurations tested consisted primarily of varying
the length of the tube connecting the pressure pick-up to the
generator. These were made using standard 1/4 inch outer diameter
0.030 inch thick copper tubing of 12, 24, 42, and 60 inches in
length. The zero length runs, to which the others were compared,
were made using a standard 1/2 inch pipe nipple which placed the
instrument pressure diaphragm about three inches from the
generator.

Tests were also made using 3/8 and 1/2 inch outer diameter
copper tubing. However, since the wave generator and the pressure
pick-up had 1/8 inch pipe fittings the end of these tubes were
restricted in diameter to use these fittings. This was accomplished
by cutting the standard half union of a flare coupling for the
larger tube in half and joining it to the 1/8 inch pipe end of a
1/4 inch tube half union.

In addition, some investigation of the effect of different
configurations of surge tanks or plenum chambers in the connecting
tube was made. The surge tanks used were standard Fram oil filters
of approximately 0.059 cubic feet volume. A single filter in an
equivalent 42 inch long tube and a π type filter utilizing two
Fram oil filters in series with an equivalent tube length of
12 inches were used.

To determine if the pressure system manifold was contributing undesired oscillations in the produced wave, a surge tank was placed between the gage pressure manifold and the generator for a series of zero tube length test runs for both the Statham and Electro-Pressuregraph gages.

Before each oscillogram was made, the zero pressure line was checked on the oscilloscope to ascertain that no vertical drift of the oscilloscope trace had occurred. An exposure of the zero pressure line was then taken. In the case of the Statham gage tests, a film exposure of the calibration pressure line was then taken by closing the switch for the calibration resistance, R_c, of Fig. 6. The switch was then opened and the trace was examined to ascertain that it had returned to the zero reference pressure. The motor was then started and the speed adjusted to that giving the approximate desired fundamental frequency of the wave. This was done by use of the strobotac. Gage pressure was then applied by opening the valve, B, of Fig. 5, until the desired pressure was read on the manometer. The wave as shown on the oscilloscope screen was then observed and synchronized and the picture was exposed. Immediately on taking the picture the pressure as shown on the manometer and the revolution speed of the generator as shown by the strobotac were read and recorded. The pressure was then turned off and the motor stopped.

The gage pressure could be maintained very steady during running, but the motor speed tended to vary plus or minus 20 RPM (0.66 cps of the fundamental wave frequency) at the higher speeds. However, the revolution speed was believed to be quite accurately determined at the time the picture was made.

RESULTS AND DISCUSSION

For each oscillogram taken of a given wave, the following items were recorded: (1) RPM of the generator shaft by use of a calibrated strobotac, (2) the gage pressure applied to the wave generator as read on the U-tube manometer, and (3) the exposures taken, for example the zero pressure line, the calibration pressure line, and the wave itself. From item (1), the RPM, the fundamental frequency of the wave in cycles per second was determined by dividing the RPM by 30. After development of the film, the frame number of the picture on the film was recorded on the data sheet for each picture to facilitate finding any desired oscillogram of a given configuration for a given frequency.

One method of analyzing the oscillograms would have been to make an enlargement of each negative and use these to take all measurements. However, the time involved for enlarging the nearly six hundred photographs taken was prohibitive, so that another method of obtaining data was decided upon.

The availability of a microfilm reader which permitted the use of 35 mm. film offered a convenient method of studying the films. By placing a uniformly ruled section of vellum graph paper in the slide, the microfilm reader was checked for linear magnification throughout the range of magnification of the instrument (from 11:1 to 23:1) and was found to be linear on all sections of the ground glass screen.

In the attempt to determine attenuation due to change in tube length, many difficulties arose in interpreting the oscillograms. As soon as any tube length was added, the wave shape became distorted to such a degree that the actual amplitude of the fundamental wave was not directly mensurable. This excessive distortion was primarily the result of the fact that a square wave is so rich in harmonics, that for any finite tube length and frequency there will be some harmonic distortion. Although the wave generated was not a perfect square wave, at the lower frequencies a very close approximation to a square wave was realized as born out by the oscillograms, (Fig. 7).

It can also be seen that even for the zero tube length configuration the wave distorted considerably at the higher frequencies. This was considered primarily a function of the generator although it was also due to limitations of the pressure sensing instrument. It can be seen that at the higher frequencies, the diametrically drilled hole in the rotating element of the generator is opened to the tank pressure (or atmosphere) for a much shorter period of time. As the port opens to exhaust to the atmosphere, for example, the entrapped gas at the tank gage pressure expands rapidly and the inertia of the expanding gas carries the enclosed pressure below atmospheric. However, before the pressure equalizes with the atmosphere the port closes, and the pressure remains below atmospheric. Actually there is some

leakage around the rotating element so that the negative gage
pressure rises slowly toward the zero gage pressure line and is
not perfectly flat. Similar reasoning can be applied for the part
of the cycle when gas at tank pressure rushes into the rotating
port. However, by comparing the oscillogram of a wave for a
finite tube length to that for zero tube length both at the same
frequency, the characteristics due to the generator itself are
eliminated.

An attempt was made to measure attenuation by integrating
the area enclosed by the wave trace about the zero pressure line.
This was accomplished by tracing the oscillogram wave on tracing
paper for each tube length at frequencies of every ten cycles per
second from 30 to 100 cycles per second. Although a long and
tedious task, the waves were accurately traced at full magnifi-
cation of 23 times through use of the microfilm reader. Full
magnification was used to ascertain that all oscillograms were
magnified the same amount. As the oscilloscope trace line at this
magnification was often quite a heavy line, 1/32 to 1/8 inch wide,
the tracing was made by following the centerline of the projected
trace. The areas were then integrated by use of a planimeter.
Each integrated area was arrived at by averaging two readings
when they varied less than one-half of one per cent. If planimeter
readings varied more than this, more readings were taken and
averaged in.

Since the wave length as shown on the oscillogram was
purely arbitrary, depending on the sweep speed and x-axis
amplification as set on the oscilloscope, the area was divided by
the wave length, λ . This gave the mean ordinate of the wave,
and, when multiplied by the static pressure calibration scale
factor, S_c, represented mean pressure.

Again attention is called to the fact that all
oscillograms, including the static pressure calibrations, naturally,
were traced at full magnification of the microfilm reader. Also,
in the case of the Statham gage, all calibration runs for the
Statham gage conveniently gave the same constant linear calibration
of one inch deflection for one inch of mercury gage pressure, see
Fig. 3 for a sample of the calibration curves. (For the Statham
gage, full D.C. amplification of the y axis on the oscilloscope was
used throughout all test runs, so that quantitative comparison
between test runs would be facilitated.)

However, it was impractical to apply the identical gage
pressure to the square wave generator for all test runs, although
they were within the same general value with the exception of a
few intentional changes in pressure amplitude to determine any
effect of pressure amplitude. Therefore, in comparing the amplitude
of a pressure wave resulting from a test run with a finite tube
length to that for the zero tube length both at the same frequency,
a correction was required to reduce both to the same impressed

pressure. See sample calculations for a hypothetical case using this method of analysis.

However, for the Electro-Pressuregraph, as previously mentioned, the calibration of the instrument was not constant throughout the test runs, see Fig. 9. To further complicate the problem, its calibration was not even linear. However, for the region used during the dynamic tests, the first calibration was essentially linear and used as such. For the second calibration of the Electro-Pressuregraph which was for the test runs of the 60 inch tube length, a straight line was drawn for the region used in the dynamic tests utilizing the theory of the least squares. However, the non-linear response of the Electro-Pressuregraph makes this type of analysis useless unless the tests are within a practically linear portion of the calibration curves.

This method of analysis utilizing the area enclosed by the wave about the zero gage pressure line proved too inaccurate to produce reliable results and was finally discarded.

The only other alternative method of analysis to obtain attenuation results which could be checked with some available theory appeared to be harmonic analysis. Unfortunately, insufficient time remained to analyze more than a few of the waves. It was decided to analyze the zero, 12, 24, 42, and 60 inch tube lengths at 30 cycles per second as obtained with the Statham gage. The Statham gage oscillograms were preferred over

the Electro-Pressuregraph tests due to the excellent linear
response and constant calibration value of this instrument which
permitted better comparison between test runs. Each oscillogram
was analyzed in the following manner. One wave length was divided
into 36 equal segments, and the ordinate at each division was
measured. The choice of 36 divisions was made because each seg-
ment then conveniently represented 10 degrees and the fact that
the analysis becomes more accurate as the number of ordinates
used is increased. From this, the first six harmonics were
determined for each oscillogram. (Only six harmonics were deter-
mined mainly due to the lack of time.) This was accomplished from
the basic formula for a Fourier cosine-sine series of:

$$p = a_0/2 + a_1 \cos t + a_2 \cos 2t + \ldots + a_n \cos nt + \ldots$$
$$+ b_1 \sin t + b_2 \sin 2t + \ldots + b_n \sin nt + \ldots$$

where the coefficients a_n and b_n are:

$$a_n = \frac{1}{p} \sum_{q=0}^{35} A_q \cos \frac{nq\pi}{p}$$

$$b_n = \frac{1}{p} \sum_{q=0}^{35} A_q \sin \frac{nq\pi}{p}$$

The item A_q is the ordinate at the qth division of the abscissa,
p is the number of divisions per half cycle, and n is the particular
harmonic. The term $a_0/2$ is the $(\sum A_q)/2p$, or the mean ordinate
over the cycle.

Table I is a listing of the coefficients for these terms for each of the tube lengths analyzed at a frequency of 30 cycles per second.

For the sake of simplicity and to enable better comparison between the different tube lengths, these in turn were resolved into the resultant or complete harmonic equations by use of the formula:

$$P = \sum (a_n^2 + b_n^2)^{\frac{1}{2}} \cos (nt - e)$$

where $e = \tan^{-1} (b_n/a_n)$

and $(a_n^2 + b_n^2)^{\frac{1}{2}}$ is the amplitude.

The values for these coefficients and phase angles, e, are listed in Table II.

Fig. 10 is a plot of the fundamental harmonic for the zero and 60 inch tube length pressure waves at a frequency of 30 cycles per second. From this it can be seen how phase lag and amplitude attenuation can be determined through harmonic analysis.

Table III is a listing of ratios of the pressure amplitude for the indicated tube length to the pressure amplitude for zero tube length where both are corrected to the same pressure amplitude at the source and with the same fundamental wave frequency of 30 cycles per second. Unfortunately, upon completion of the analysis it was discovered that the zero tube length oscillogram was one for a reduced pressure at the source of 3.74 inches of

mercury gage, whereas the others were in the vicinity of 5.70 inches
of mercury. All cases were corrected to 5.70 inches of mercury
gage pressure. In this manner the attenuation in pressure amplitude
for any desired harmonic can be determined.

In addition to attenuation, the phase shift of any desired
harmonic compared to the same harmonic for the zero tube length
case can be obtained by merely subtracting the phase angle, e_{ol},
for the zero tube length from the phase angle, e_1, for the finite
tube length, both for the same harmonic and basic fundamental
frequency.

Furthermore, the resultant harmonic equations give a
mathematical indication of distortion. Examination of Figs. 11a
and 12 shows some harmonics practically damped out whereas others
are magnified as tube length is increased.

Unfortunately, insufficient time prevented more harmonic
analysis at other frequencies or to higher harmonics as well as for
some of the other configurations tested. For the tube lengths
tested, harmonic analysis at a higher frequency, 60 or 90 cycles
per second, undoubtedly would have given a much better check with
existing theory.

However, Fig. 11a is a plot of the ratio of pressure
amplitude for finite tube length over zero tube length pressure
amplitude at 30 cycles per second as obtained by experiment and
through use of the theoretical graphs prepared by A. S. Iberall

in reference (4). Attention is called to the fact that this is

for the fundamental harmonic only. For the tube lengths and

frequency analyzed, since the variations in attenuation were of

such small magnitude and considerable interpolation in the use

of the theoretical curves was necessary, the degree of agreement

between experimental and theoretical values cannot be necessarily

assured. However, they appear to be in fair agreement, and with-

out more experimentally determined points the author cannot

ascertain agreement. Fig. 12 is a plot of the pressure amplitude

for finite tube lengths to amplitude for zero tube length for the

second through sixth harmonics obtained experimentally.

Fig. 11b illustrates the comparison of phase lag with

tube length compared with that obtained from the theoretical graphs

of reference (4) for the fundamental harmonic at 30 cycles per

second. The apparently poor agreement in phase lag might largely

be attributed to interpolation errors in the theoretical graphs

and the determination of the phase angle, e, in the resultant

harmonic. A slight change in the value of b_1/a_1 changes the angle

e considerably. The angle of lag, $\emptyset$, for the 42 inch tube length

was plotted as 15° lead in lieu of 345° lag to facilitate plotting.

Attention is called to the fact that the pressure ratio was also

shown to be greatest for the 42 inch tube length, but since this

was considerably below the resonant frequency of approximately 75

cycles per second, the point should be viewed with considerable

suspicion.

An attempt to determine phase lag directly from the oscillograms was done by magnifying each cycle to an arbitrary linear scale of 36 major divisions representing 360 degrees. Then by placing zero of this scale on the timer dot on the oscillogram representing the start of the wave cycle at the generator and reading the value on the scale which was aligned with the initial pulse of the gage pressure trace starting the cycle on the oscillogram, the lag could be read directly. However, in practice the phase lag was not so easily determined. As soon as any tube length was added and as the frequency was increased, the square pressure wave was distorted enough so that determination of the initial pulse of tank pressure was often impossible and required personal estimation. Fig. 13 is offered to illustrate the high degree of scatter of points for the plot of phase lag, ϕ , against frequency for the tube lengths tested, and as obtained directly from the oscillograms utilizing the timer marks.

Attention is called to the fact that a flexible coupling consisting of a stiff rubber hose was used to drive the square wave generator. Considerable torsional vibration was present since the pulse of pressure just as the valve ports opened or closed caused a change in the torque load required to rotate the wave generator by the application of a tangential component of jet thrust. This excited a forced torsional vibration which was apparent on most of the oscillograms by the fact that successive wave lengths were not always the same as well as the fact that the top half of the wave

was usually of different length than the lower half. As the tests
progressed the hose coupling appeared to suffer considerable wear
and lost much of its stiffness. Thus it was feared that the amount
of lag due to twist in the coupling had increased considerably over
that for previous tests. Consequently, a universal joint was
substituted for the test runs in which the Electro-Pressuregraph
was used. This reasoning appears to be verified by Fig. 13 where
it can be noted that the phase lag for the 60 inch tube length
(obtained from the Statham gage runs) is considerably greater than
that obtained from the Electro-Pressuregraph tests in which the
universal joint was used. The 60 inch tube tests for the Statham
gage were the last test runs performed using the rubber hose
coupling.

In the determination of phase lag, it was assumed that
phase lag for the zero tube length test runs was zero at all
frequencies tested which was practically true as indicated on the
oscillograms. Actually at the higher frequencies a lag of five to
ten degrees was indicated on the zero tube length oscillograms and
was credited to additional twist in the hose coupling. Thus the
position of the timer dot on the oscillogram for a given tube length
at a given frequency was compared to the position of the timer dot
for the same frequency of the zero tube length oscillogram.

An attempt to check the attenuation and phase shift
obtained through harmonic analysis with that obtained utilizing

the analogy of a R-L-C electrical circuit as given in reference (3) was made. Again, however, due to the few test runs analyzed and the relatively low frequency for the tube lengths used, the results were inconclusive and are therefore not presented in this report.

Fig. 14 is a reprint from reference (3) of the pressure response and phase lag of a sinusoidal oscillating pressure of small amplitude for various tube lengths given in wave lengths, λ , where λ = cf. These curves are obtained theoretically from the solution of A. C. theory equations for a R-L-C circuit analogous to a pressure sensing system of negligible instrument volume. The effect of instrument volume is to shift the pressure response curves to the left and to decrease the ratio P_r/P_s below the value of unity.

No specific analysis of the larger diameter tubes or of the plenum chamber configurations was conducted. However, just through observation of the oscillograms, the effect of the larger diameter tubes as used in this investigation was to decrease the amplitude of the oscillation much like a small surge tank. In reality this is what was actually present in this configuration since the end fittings remained the same small diameter used with the 1/4 inch O.D. copper tubes.

Similarly, by inspecting the oscillograms for the surge tank configurations it was noted that the π type filter gave the greatest damping of the pressure wave. It appeared that, as would be expected from theory, that little more than the fundamental

frequency was passed through the filter and this was damped greatly

itself. The π type filter is similar to a low frequency pass

filter which filters out higher frequencies and permits low

frequencies to pass through.

CONCLUSIONS AND RECOMMENDATIONS

From the results of this investigation of the environ-
mental effects on pressure sensing systems for oscillating pressures,
the following conclusions have been made:

(1) Any arbitrarily produced pressure wave may be used
in the determination of attenuation, phase lag, and distortion
effects due to the pressure sensing configuration if the wave
observed resulting from a specific configuration is compared to the
wave obtained with the pressure sensing instrument connected directly
to the source of pressure oscillations when both cases are at exactly
the same frequency and impressed pressure amplitude. It should not
necessarily be construed that the change in amplitude and phase
angle of the fundamental of such an arbitrary wave containing higher
harmonics would be identical to the change in amplitude and phase
angle for a pure sinusoidal oscillation under the same conditions.
This is a question which well might warrant further experimental
investigation.

(2) Harmonic analysis offers the best method in the
determination of attenuation, phase lag, and distortion resulting
from the tube connecting a pressure sensing instrument to the
source of oscillating pressure. All three of these effects are
obtainable from harmonic analysis.

(3) An increase in tube diameter anywhere between the
ends of the tube connecting the pressure sensing instrument to the
source of oscillation acts as a small surge tank which decreases
the amplitude of oscillation.

(4) For the frequency range investigated a π type filter arrangement of two surge tanks was the most effective method of damping out pressure oscillations.

It is recommended that if further investigations are made utilizing the same square pressure wave generator, the use of an harmonic analyzer might be used to good advantage enabling the use of more oscillograms of waves produced over a wide range of frequencies. The addition of one or more longer tube lengths is also highly recommended. A tube length of 10 feet would permit a convenient comparison with the theoretical results obtained using the R-L-C circuit theory analogy of reference (3), which contains a table of calculated pressure response ratios for various frequencies for a 10 foot long tube.

The ideal pressure wave for an investigation of this type is naturally a sine wave. Any distortion appearing in the pressure wave would be indicated then by the appearance of higher harmonic terms which could be determined through harmonic analysis of the oscillograms obtained. The difficulty of designing a sinusoidal pressure wave generator is common knowledge. However, there are several ways in which a wave of approximately sinusoidal oscillation can be generated. For a generated pressure wave of approximately sinusoidal oscillation, the analysis would still be greatly simplified due to the lesser importance of higher harmonics in the initial wave generated.

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APPENDIX

SYMBOLIZATION

A = Area—tube cross sectional or of integrated oscillogram

D = Inside diameter of tube

L = Tube length

S_c = Pressure calibration scale factor

V = Instrument volume

a_n = n^{th} harmonic coefficient of cosine term in Fourier series

b_n = n^{th} harmonic coefficient of sine term in Fourier series

c = Propagation velocity

e = Phase angle

f = Frequency

p = Pressure

t = Time

z = Dimensionless parameter characterizing damping

λ = Wave length

ϕ = Phase shift or lag

χ = Attenuation factor

μ = Fluid viscosity

$\rightarrow$ = Kinematic viscosity

ω = Angular frequency

ξ = Fractional pressure excess

SAMPLE ANALYSIS OF A HYPOTHETICAL CASE
FOR THE OSCILLOGRAMS OF TWO SQUARE
WAVES UTILIZING THE INTEGRATED AREA

ZERO TIME LENGTH   FINITE TIME LENGTH
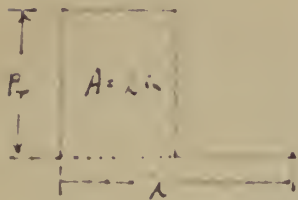
$P_S = 2.1$ in H_3 (SUPPLIED TO SOURCE) $P_S = 2.0$ in H_3

$P_{r_0} = 2.0$ in H_3 (READ ON OSCILLOGRAM) $P_{r_c} = 2.0$ in H_3

$\lambda = 2.0$ in. (WAVE LENGTH) $\lambda = 2.0$ in.

S_c = CALIBRATION SCALE 1 in H_3/in. $S_c = 1$ in H_3/in

$$\frac{A S_c}{\lambda P_S} = \frac{(2.0)(1.0)}{(2.0)(2.1)} = 0.476$$ $$\frac{A S_c}{\lambda P_c} = \frac{(2)(2.1)}{(2.0)(2.0)} = 0.400$$
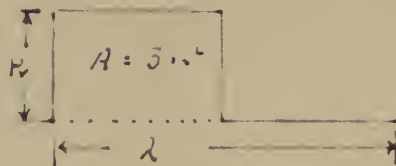
TAKING RATIO OF THESE TWO DIMENSIONLESS PARAMETERS

REPRESENTING A MEAN PRESSURE CALCULATE:

$$\frac{P_{r_c}}{P_{r_c}} = \frac{0.400}{0.476} = 0.91 \sim$$

CHECKING BY DIRECT RATIO OF PRESSURE

AMPLITUDES CORRECTED TO SAME PRESSURE AT SOURCE, P_S:

$$\frac{P_{r_c}}{P_{r_c}} = \frac{P_{r_c}}{P_{r_c}} \frac{P_{S_L}}{P_{S_L}} = \frac{(2.0)(2.1)}{(2.0)(2.0)} = 0.91 \sim$$

SAMPLE CALCULATION FOR THEORETICAL DETERMINATION
OF ATTENUATION AND PHASE SHIFT IN OF ISOLATING
FOR FUNDAMENTAL HARMONIC AT 30CPS FUNDAMENTAL FREQUENCY.

$$z = \frac{D^2}{4} \frac{\omega}{\nu_o} \quad , \quad A \text{ DIMENSIONLESS PARAMETER}$$

WHERE D = TUBE INNER DIAMETER

ω = ANGULAR FREQUENCY

ν_o = KINEMATIC VISCOSITY

$$\therefore, z = \frac{(.19)^2 (2.54)^2 (60\pi)}{4 \quad (1/6)} = 65.9$$

$$\frac{\omega L}{c} = \frac{(60\pi)(5)}{1100} = 0.855 \quad A \text{ DIMENSIONLESS PARAMETER}$$

where L = TUBE LENGTH

C = SONIC VELOCITY

$$A = \frac{\pi D^2}{4} = \frac{(\pi)(.19)^2}{4} = 0.0284 \text{ in}^2 \text{ (Tube Cross Sectional Area)}$$

ATTENUATION FACTORS:

$$K_{To} = \frac{32 \mu_o}{P_o} \omega \left(\frac{L}{D}\right)^2 \quad \text{where } \mu_o = \text{fluid viscosity}$$

$$P_o = \text{mean pressure}$$

$$= \frac{(32)(2 \times 10^{-4})(60\pi)}{10^6} \left(\frac{5}{.19}\right)^2 = 0.120$$

$$\frac{K_{io}}{K_{To}} = \frac{1}{m} \left(\frac{V}{AL}\right) = \frac{1}{1} \left(\frac{0.1}{.0284}\right)\frac{1}{5} = 0.0586 \quad \text{Where } V = \text{Instrument Volume}$$

$$m = \text{Polytropic Exp}$$

of Expansion

in Instrument

ENTERING INDICATED GRAPHS IN REFERENCE
WITH THESE CALCULATED PARAMETERS
THE FOLLOWING IS OBTAINED:

$$\left|\frac{\tilde{\xi}_{oL}}{\tilde{\xi}_{g_o}}\right|_o$$

FROM

$z \leq 1$	FIG. 2	$\rightarrow$	1.00
$z = 6.25$	FIG. 4	$\rightarrow$	1.05
$z \geq 100$	FIG. 6	$\rightarrow$	1.50

INTERPOLATED FOR
$z = 65.9$

$$\therefore \quad \left|\frac{\xi_{oL}}{\xi_{g_o}}\right|_o = 1.41$$

PHASE LAG δ_o - DEGREES

FROM: $z \leq 1$ FIG. 3	$5°$
$z = 6.25$ FIG. 5	$5°$
$z \geq 100$ FIG. 7	$50°$

INTERPOLATED FOR
$z = 65.9$, GIVES

$$\delta_o = 40°$$

TABLE I

COEFFICIENTS FOR THE FIRST SIX HARMONICS OF A GENERATED SQUARE

PRESSURE WAVE AT 30 CYCLES PER SECOND AS OBTAINED FOR VARIOUS TUBE LENGTHS

$$p = a_o/2 + \sum a_n \cos nt + \sum b_n \sin nt$$

Coefficient	Tube Length, Inches				
	0	12	24	42	60
$a_o/2$	2.870	-2.8513	3.004	1.908	1.941
a_1	-1.028	-1.110	-1.004	-0.3065	-2.578
a_2	-0.054	-0.235	1.395	0.838	-0.075
a_3	-0.524	-0.800	-1.647	-0.947	0.352
a_4	0.122	1.133	-0.688	60.021	-0.045
a_5	-0.610	-0.932	-0.323	-0.926	0.342
a_6	0.168	-0.741	-0.242	-0.261	0.083
b_1	3.642	3.744	3.926	5.195	3.568
b_2	-0.225	-0.670	-0.835	-0.132	1.286
b_3	0.772	1.065	0.051	0.177	-0.003
b_4	0.056	0.283	1.492	0.156	0.057
b_5	0.325	-0.273	-0.239	0.041	0.536
b_6	-0.052	0.624	0.513	0.114	0.130

TABLE II

COEFFICIENTS AND PHASE ANGLES FOR

FIRST SIX HARMONICS OF A GENERATED SQUARE PRESSURE

WAVE FOR VARIOUS TUBE LENGTHS AT 30 CPS

$$p = a_0/2 + \sum (a_n^2 + b_n^2)^{\frac{1}{2}} \cos (nt-e)$$

AMPLITUDE, $(a_n^2 + b_n^2)$					
	Tube Length, Inches				
Harmonic, n	0	12	24	42	60
1	3.785	3.905	4.050	5.200	4.400
2	0.232	0.710	1.670	0.848	1.2875
3	0.884	1.320	1.647	0.963	0.352
4	0.135	1.169	1.642	0.157	0.052
5	0.691	0.971	0.407	0.926	0.636
6	0.555	0.826	0.567	0.236	0.152

Phase Angle, o					
	Tube Length, Inches				
Harmonic, n	0	12	24	42	60
1	105°45'	106°31'	104°21'	91°20'	125°52'
2	256°37'	250°41'	33°18'	351°02'	93°21'
3	124°11'	126°55'	178°13'	169°24'	359°30'
4	24°44'	14°03'	114°45'	97°30'	128°06'
5	151°55'	196°18'	143°34'	177°29'	57°28'
6	342°47'	130°55'	115°14'	156°19'	57°21'
Mean Ordinate $a_0/2$	2.870	2.851	3.004	1.908	1.941

TABLE III

RATIOS OF PRESSURE AMPLITUDE FOR FINITE TUBE LENGTHS TO

THE PRESSURE AMPLITUDE OBTAINED AT ZERO TUBE LENGTH (P_L/P_{OL})

FOR THE FIRST SIX HARMONICS OF A GENERATED SQUARE PRESSURE WAVE

AT 30 CPS. PRESSURE SUPPLIED TO THE WAVE GENERATOR CORRECTED

TO 5.70 INCHES OF MERCURY FOR ALL TUBE LENGTHS

$$P_L/P_{OL}$$

Tube Length Inches	$(P_L/P_{OL})_{mean}$	Harmonic					
		1	2	3	4	5	6
12	1.005	1.030	3.060	1.496	8.650	1.405	1.488
24	1.040	1.070	7.200	1.865	12.175	0.588	1.020
42	0.665	1.372	3.650	1.090	1.162	1.340	0.516
60	0.677	1.162	5.550	0.399	0.387	0.920	0.274

TABLE IV

PHASE LAG, ϕ , FOR VARIOUS TUBE LENGTHS AS COMPARED TO

ZERO TUBE LENGTH FOR THE FIRST SIX HARMONICS OF A GENERATED

SQUARE PRESSURE WAVE AT 30 CPS

PHASE LAG, ϕ
DEGREES

Tube Length Inches	Harmonic					
	1	2	3	4	5	6
12	8	0	2	350	45	148
24	0	0	54	90	352	133
42	345	137	45	73	26	174
60	20	197	135	104	166	75
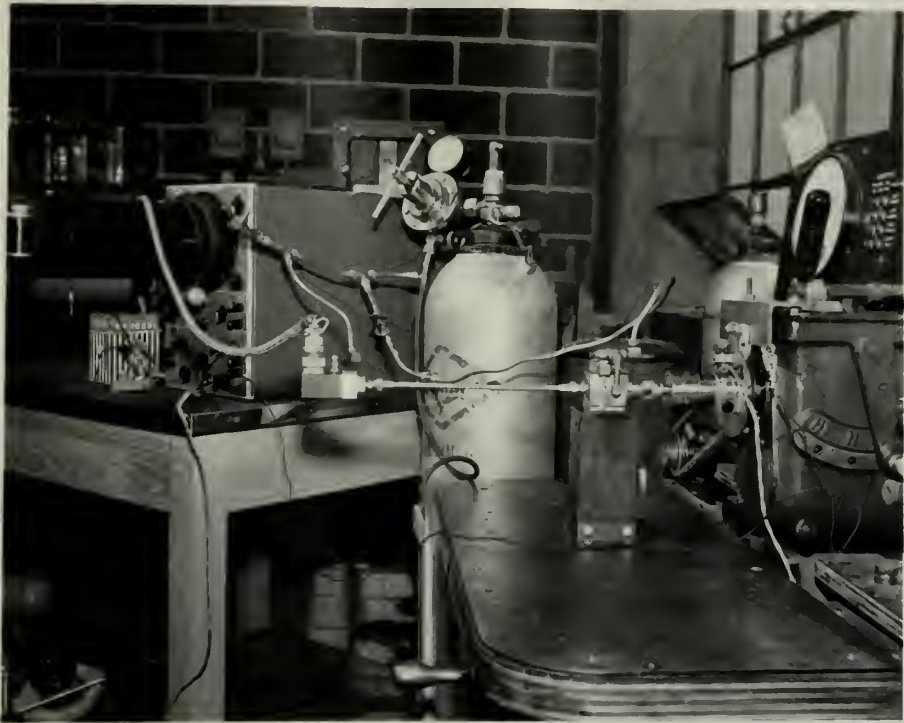
Fig. 1

Test Apparatus for Statham Gage
with 12 Inch Tube Length
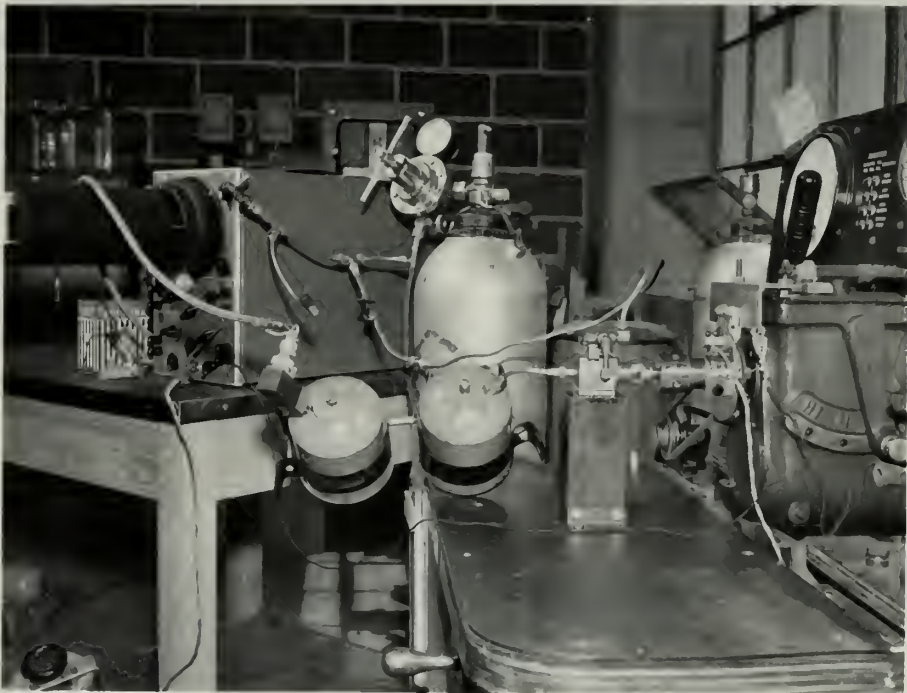
Fig. 2

Test Apparatus for π Type
Filter with Statham Gage
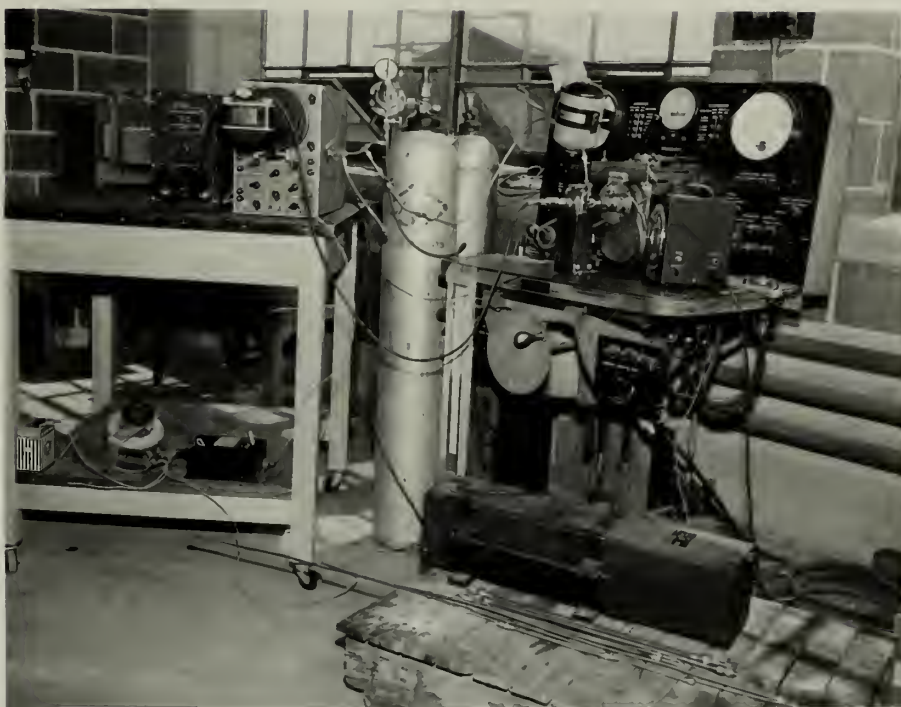
Fig. 3

Test Apparatus for Zero Tube Length
with Electro-Pressuregraph
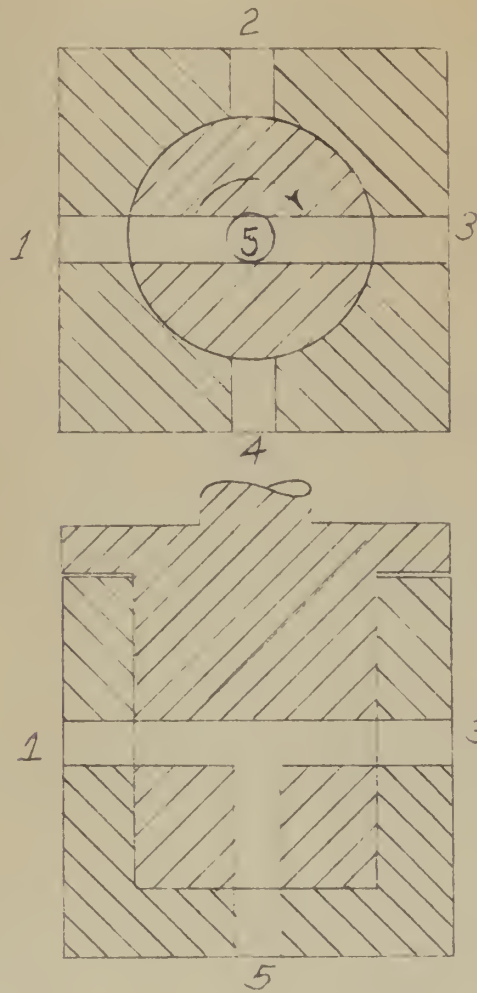
PORTS: 1-3 TANK PRESSURE, 2-4 ATMOSPHERIC PRESSURE
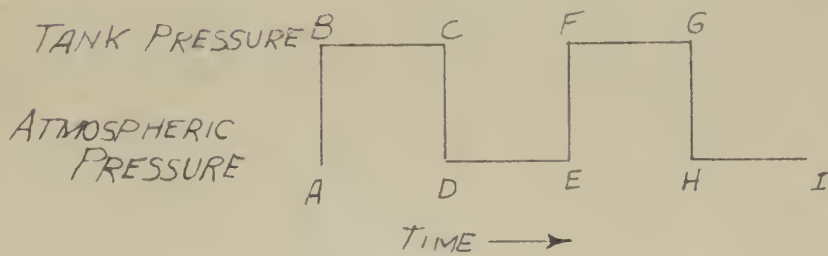
FIG. 4

SQUARE WAVE PROPAGATION
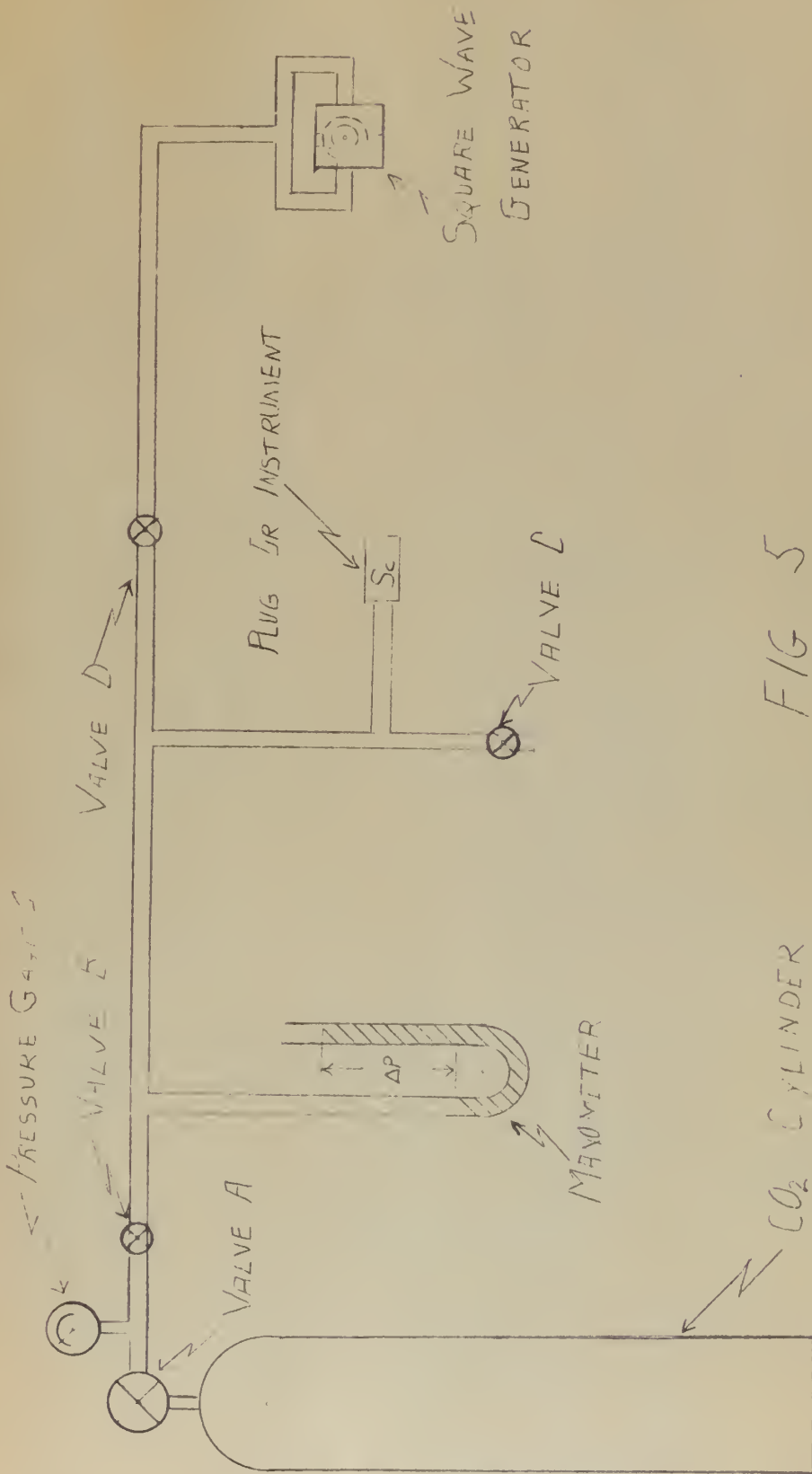
PRESSURE SYSTEM FOR CALIBRATION AND WAVE GENERATION

FIG 5

BRUSHES

PLASTIC

MOTOR

BRASS

BRUSH ROTOR

SQUARE WAVE GENERATOR

FIG 6

OSCILLOSCOPE

TRANSFORMER

S

Rc
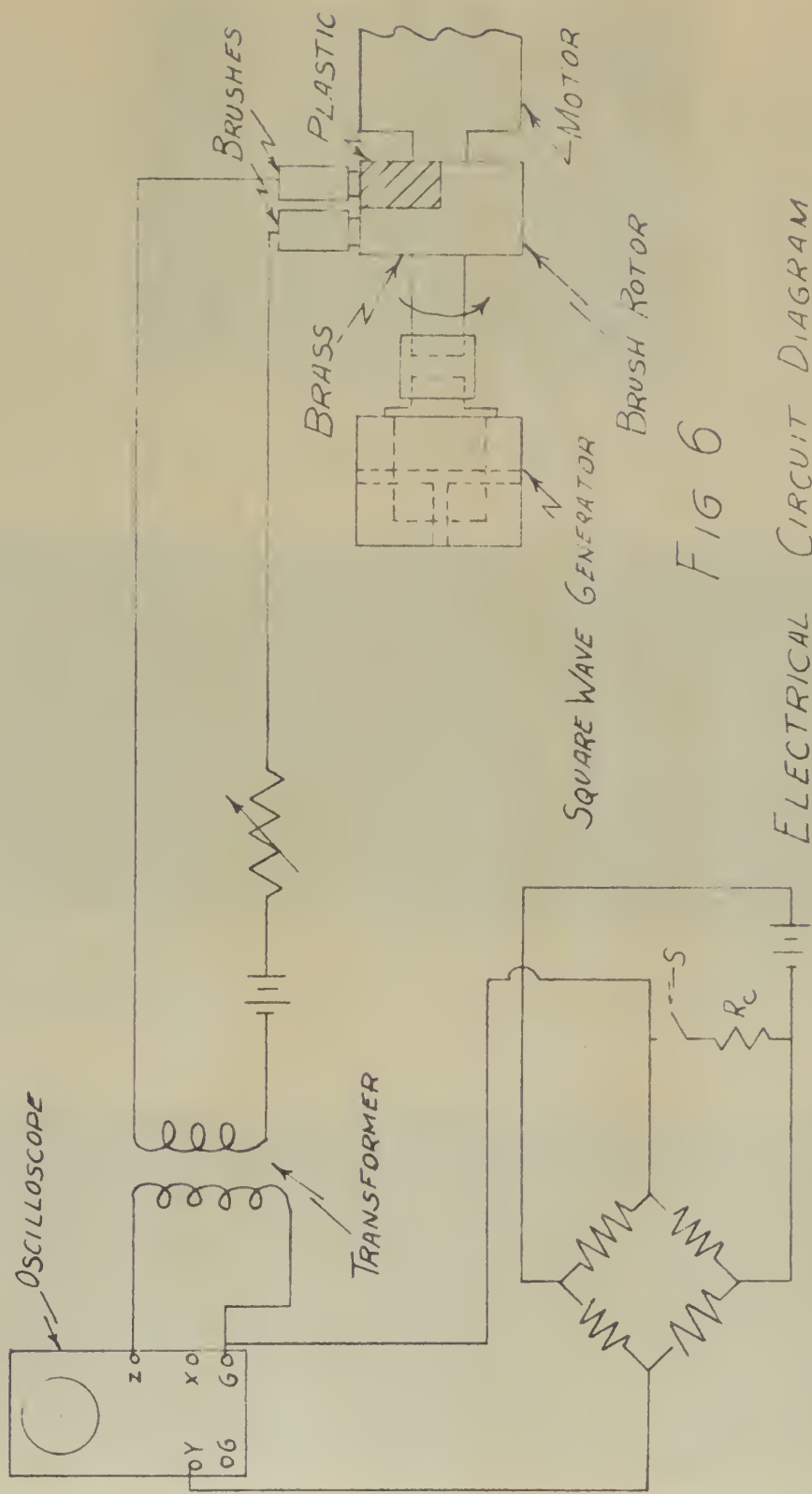
ELECTRICAL CIRCUIT DIAGRAM

STATHAM GAGE BRIDGE CIRCUIT
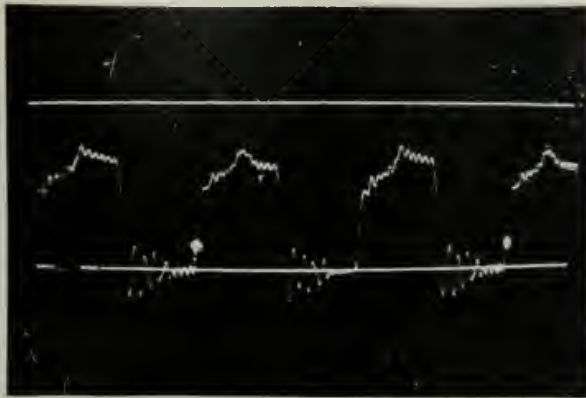
Zero Tube Length
30.0 CPS
p_s = 3.74 Inches of Mercury, Gage
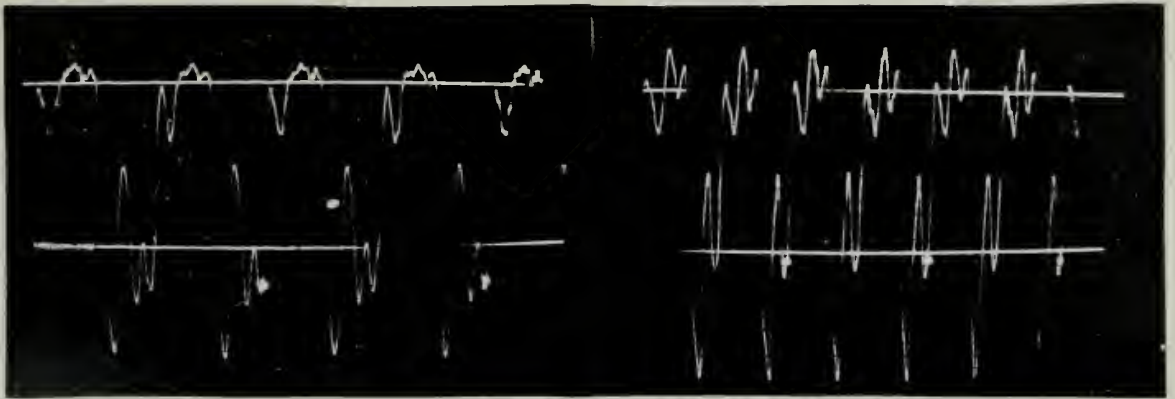
12 Inch Tube Length   24 Inch Tube Length
30.3 CPS   30.3 CPS
p_s = 5.70 Inches of Mercury, Gage p_s = 5.70 Inches of Mercury, Gage
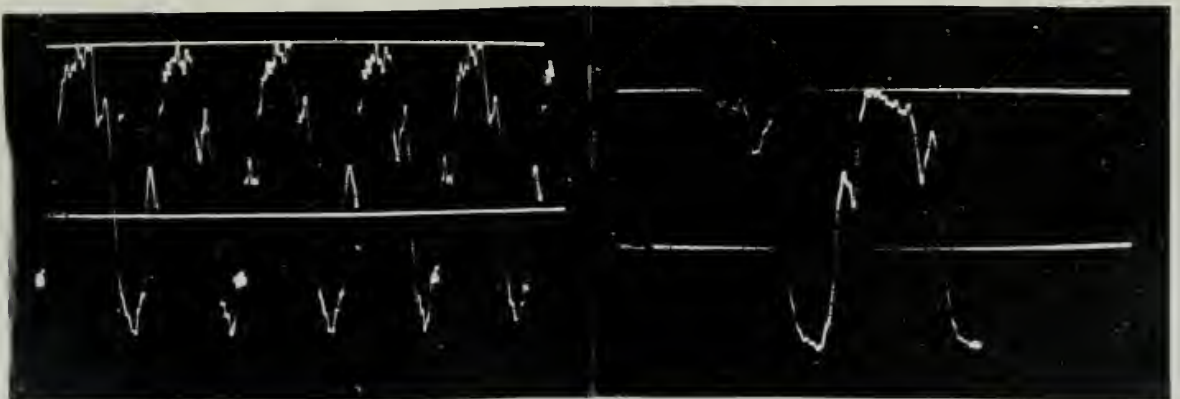
42 Inch Tube Length   60 Inch Tube Lengths
29.8 CPS   30.6 CPS
p_s = 5.70 Inches of Mercury, Gage p_s = 5.76 Inches of Mercury, Gage

Fig. 7a
Oscillograms Used in Harmonic Analysis Obtained with
Statham Gage at 30 CPS for Various Tube Lengths
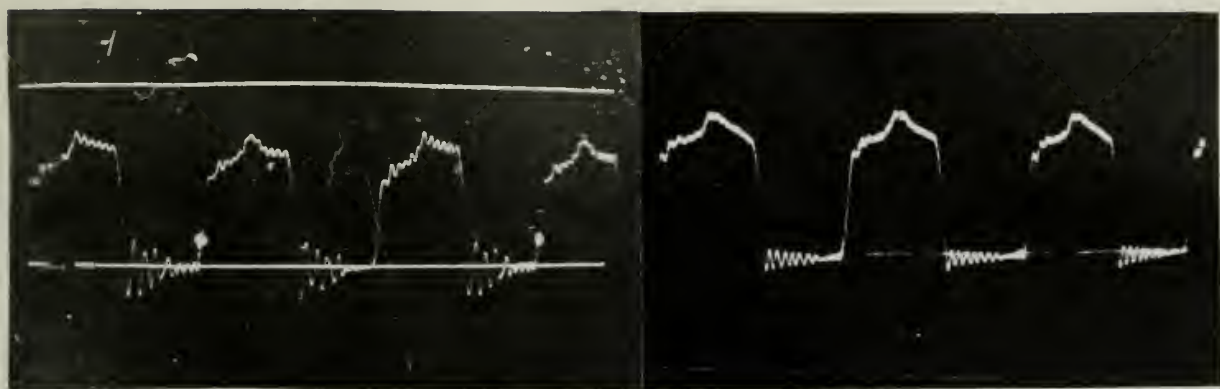
30.0 CPS   30.0 CPS
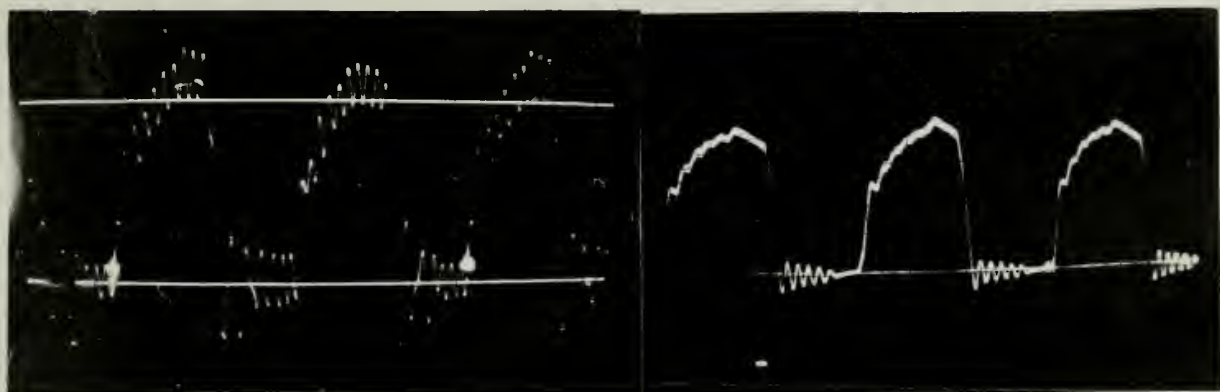
41.1 CPS   40.3 CPS
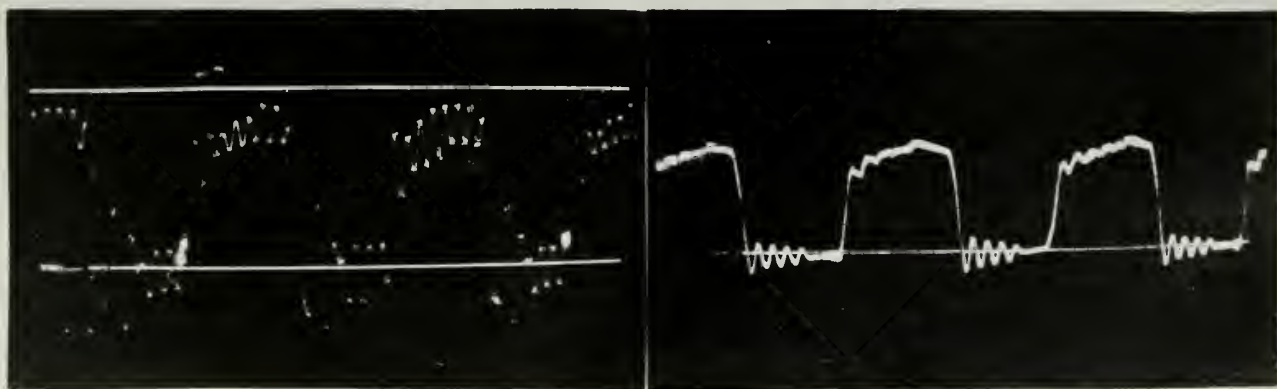
52.0 CPS   50.5 CPS

Fig. 7b
Comparison between Oscillograms Obtained
with Statham Gage and Electro-Pressuregraph
with Zero Tube Length at Various Frequencies

Statham Gage   Electro-Pressuregraph
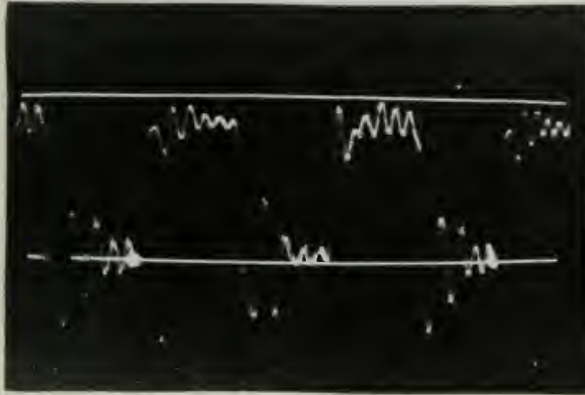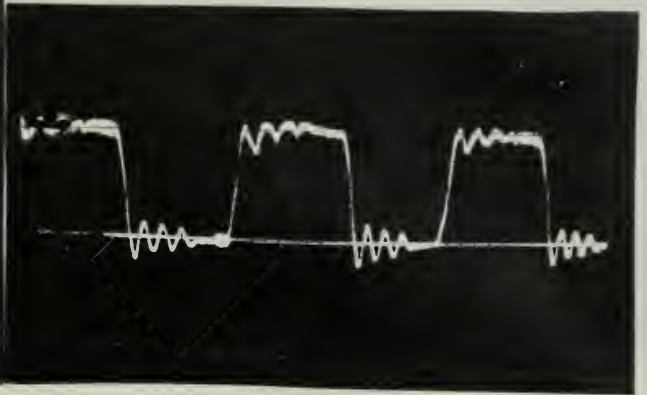

60.0 CPS   60.0 CPS
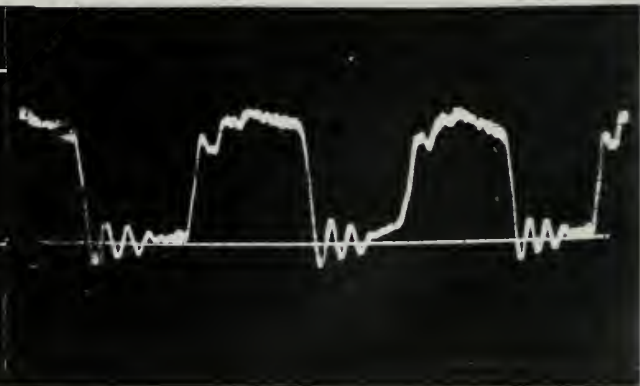
68.6 CPS   70.0 CPS

Fig. 7c

Comparison between Oscillograms Obtained
with Statham Gage and Electro-Pressuregraph
with Zero Tube Length at Various Frequencies

Statham Gage   Electro-Pressuregraph
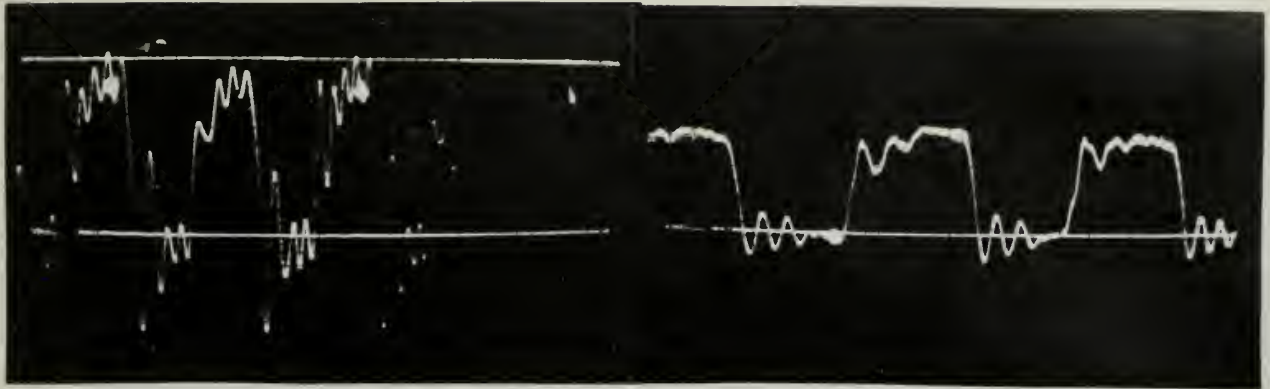
76.6 CPS   79.0 CPS
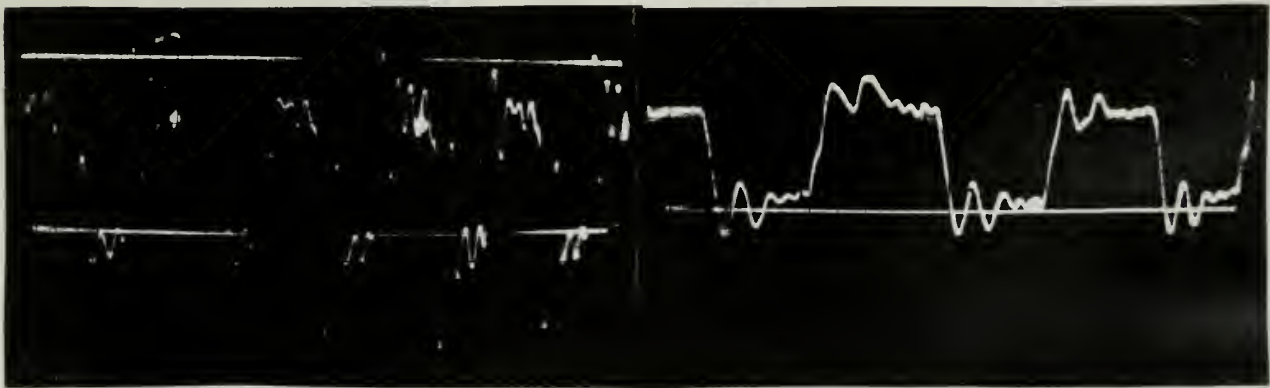
39.8 CPS   99.6 CPS

Fig. 7d

Comparison between Oscillograms Obtained
with Statham Gage and Electro-Pressuregraph
with Zero Tube Length at Various Frequencies
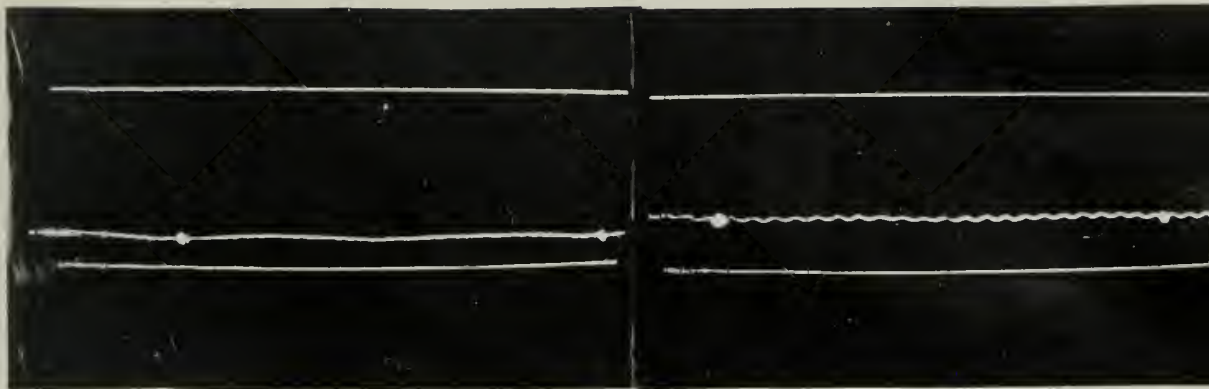
42.3 CPS   66.6 CPS

One Filter, 0.059 Cubic Feet
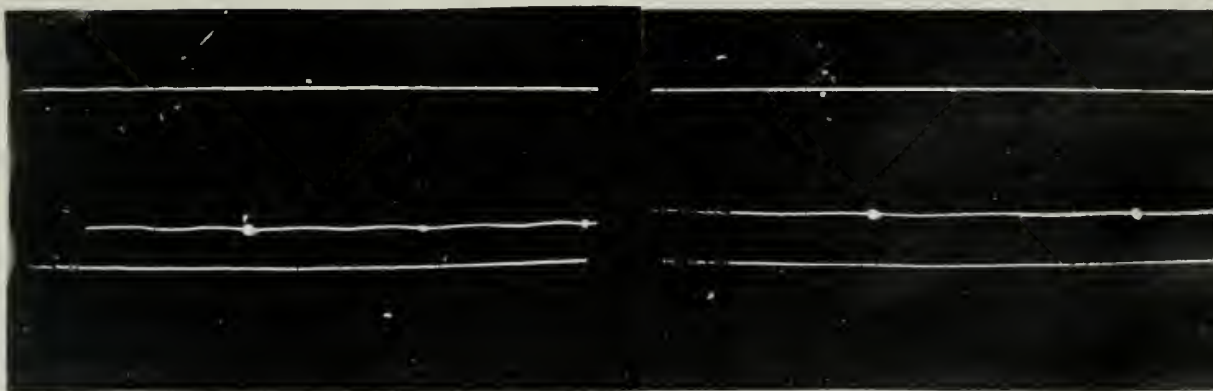
40.8 CPS   65.0 CPS

Two Filters, π Type, 0.059 Cubic Feet Each

Fig. 7e

Comparison between a Single Surge Tank
Filter and a π Type Filter for Two Frequencies
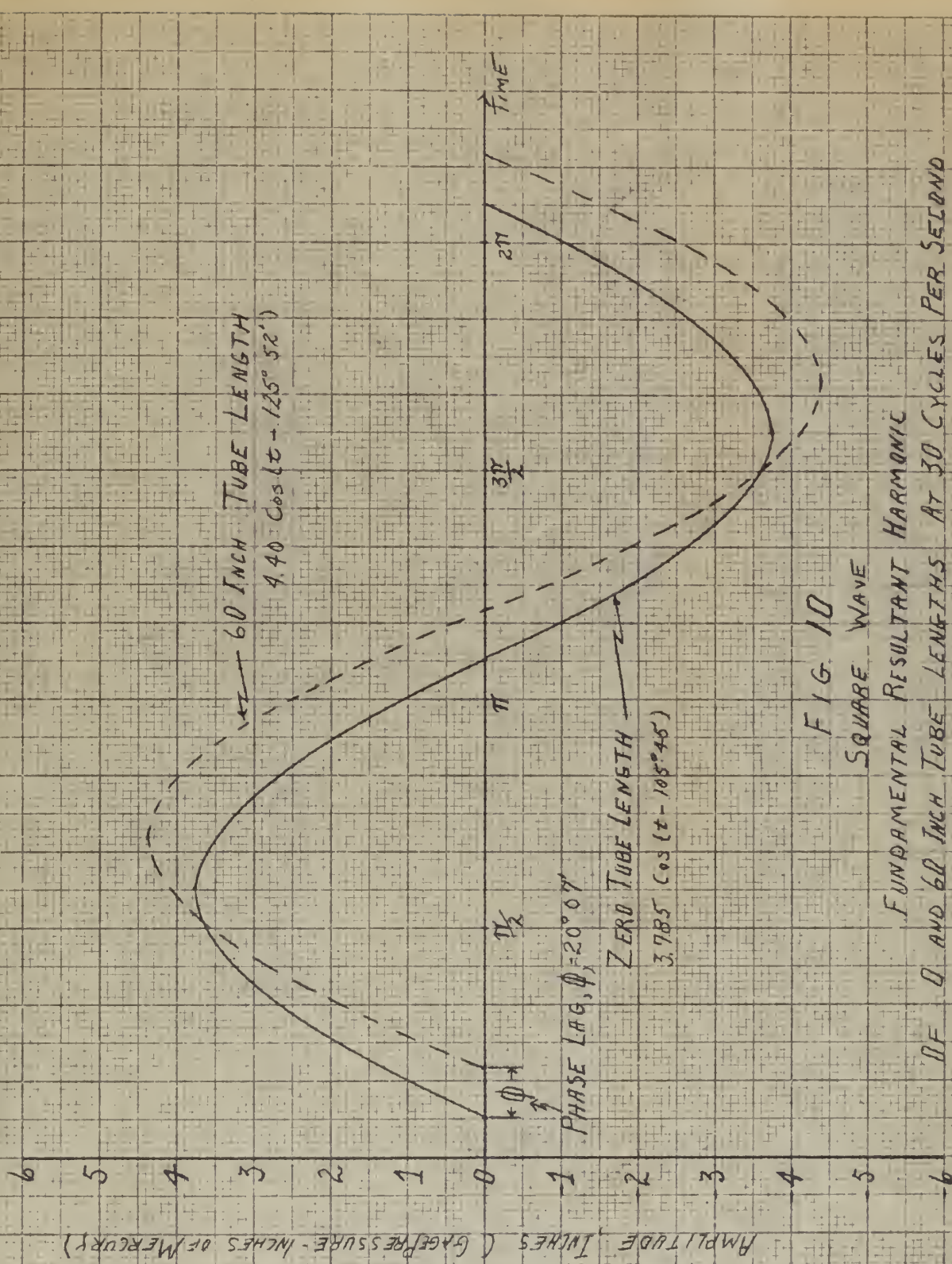p_a = 5.70 Inches of Mercury, Gage

LEGEND
⊙ MAY 3, 1952
⊡ MAY 6, 1952
△ } MAY 7, 1952
▽

FIG. 8
STATIC PRESSURE CALIBRATION
FOR STATHAM GAGE

GAGE PRESSURE, INCHES OF MERCURY

PRESSURE LINE DEFLECTION, INCHES
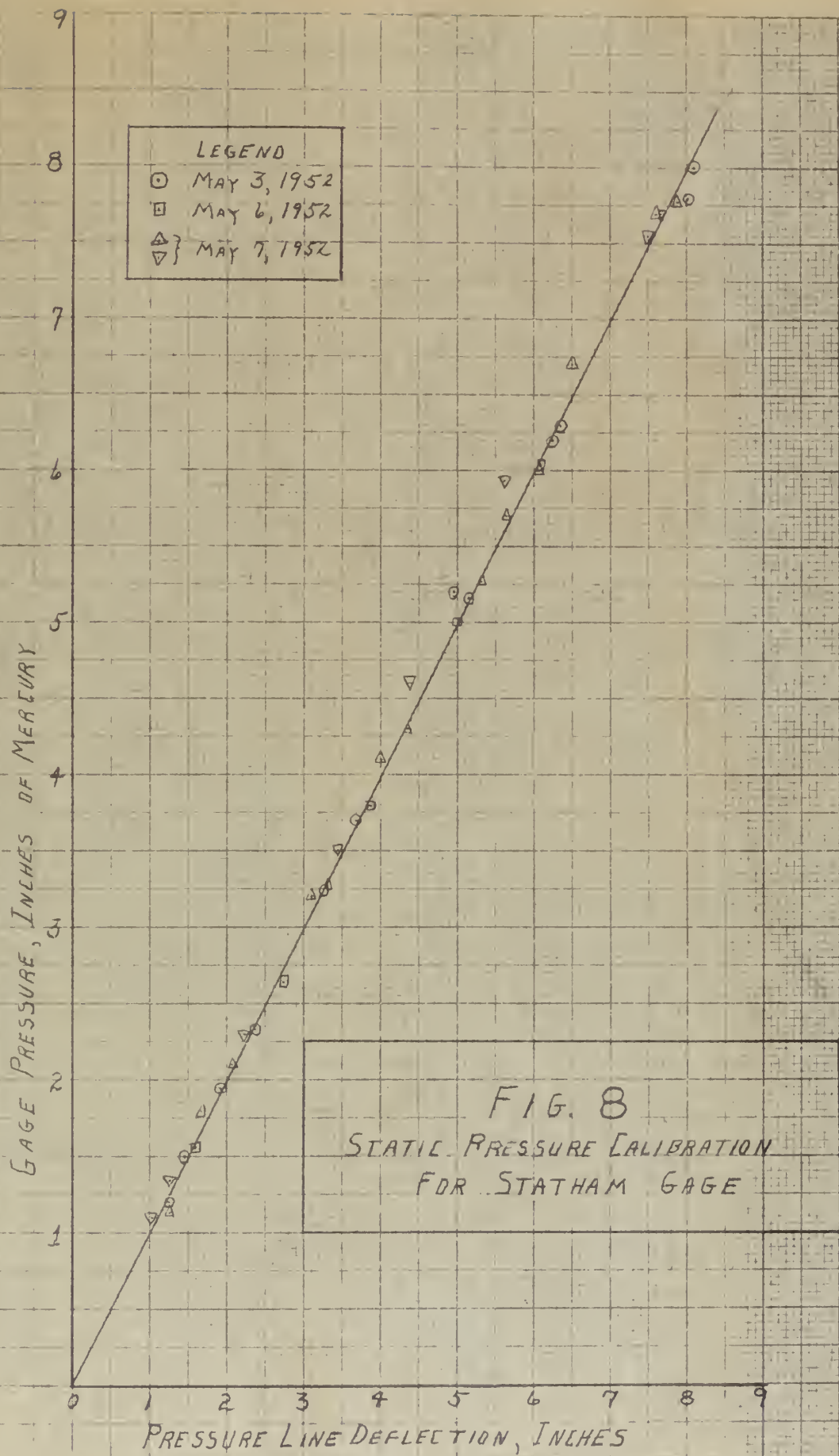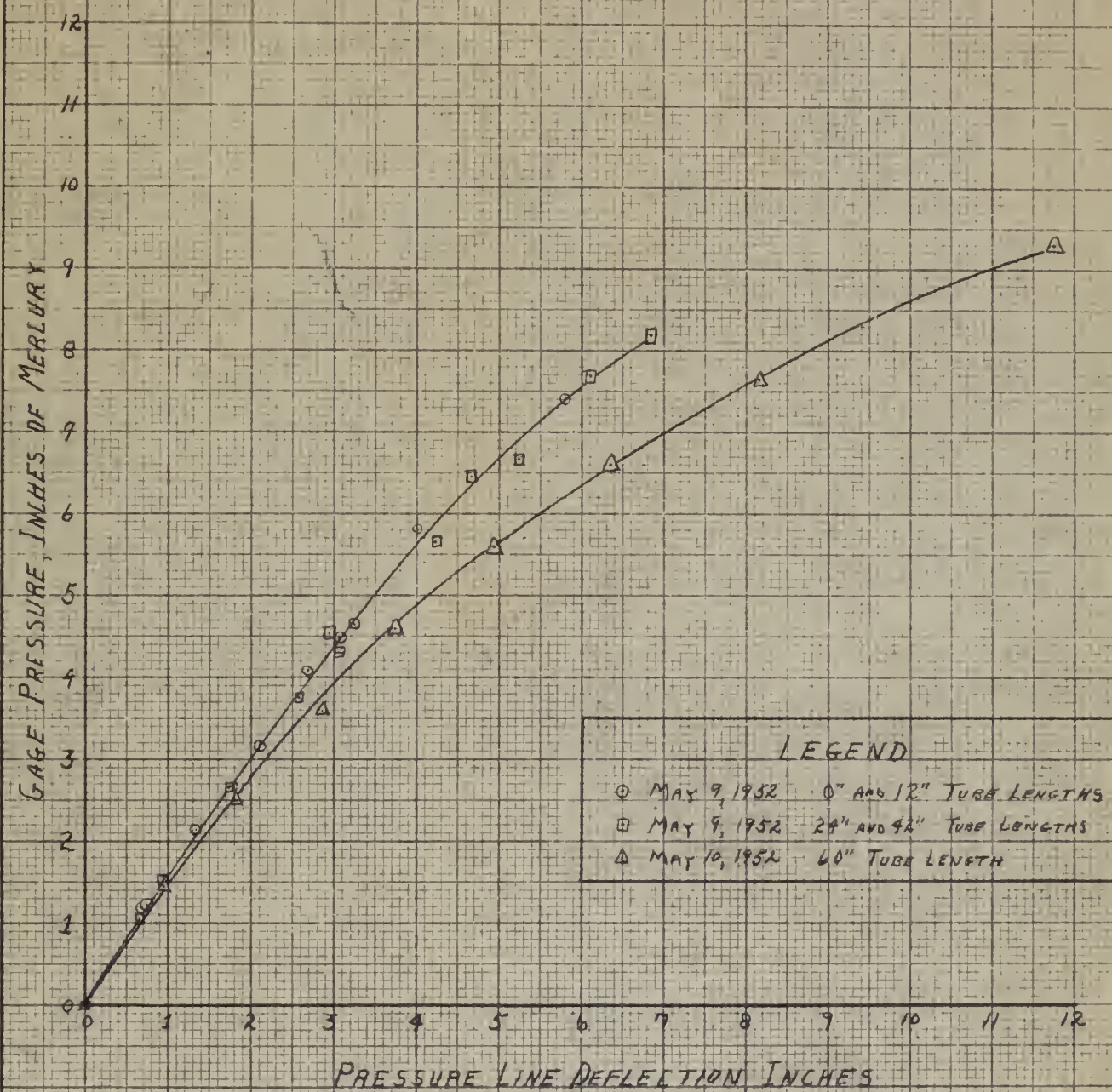
FIG. 9

STATIC PRESSURE CALIBRATION
FOR ELECTROPRESSUREGRAPH

FIG. 10

SQUARE WAVE

FUNDAMENTAL RESULTANT HARMONIC

OF O AND 60 INCH TUBE LENGTHS AT 30 CYCLES PER SECOND
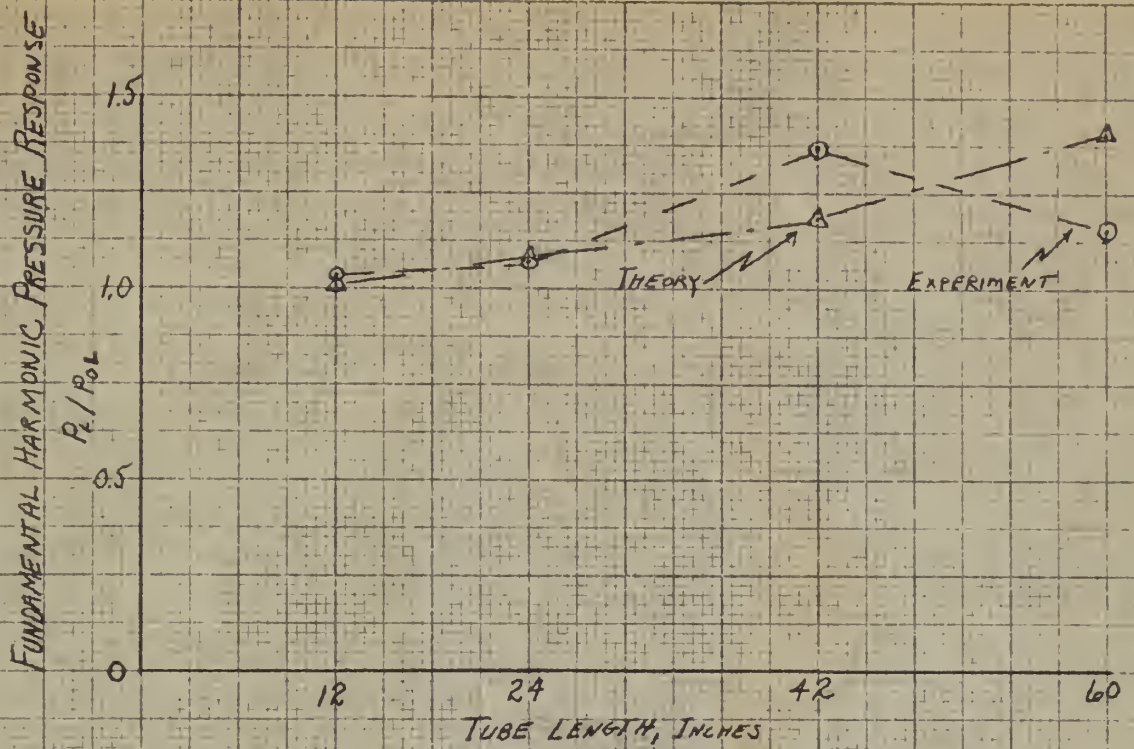
FIG. 11a

RATIO OF FUNDAMENTAL HARMONIC PRESSURE AMPLITUDE
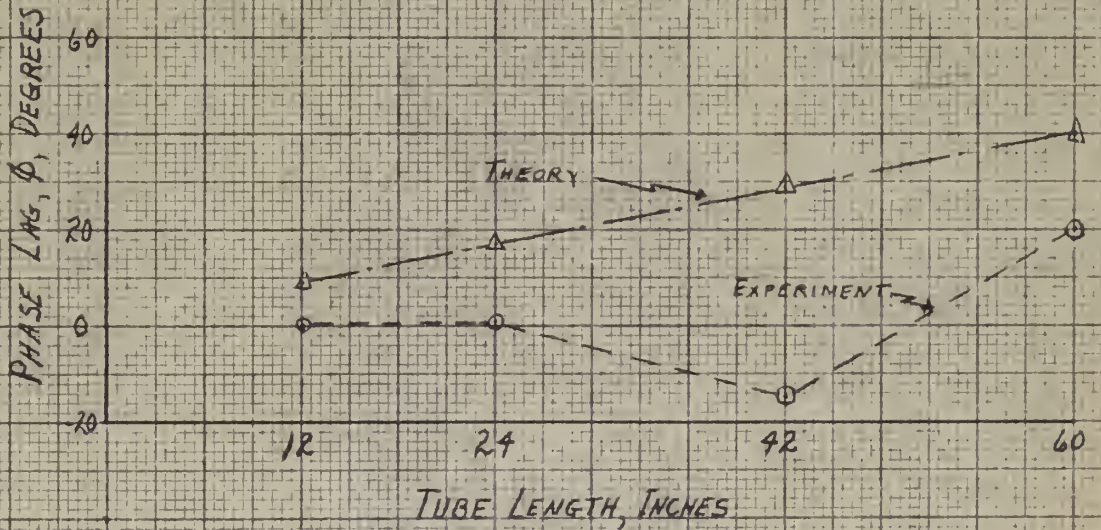FOR FINITE TUBE LENGTH TO PRESSURE AMPLITUDE
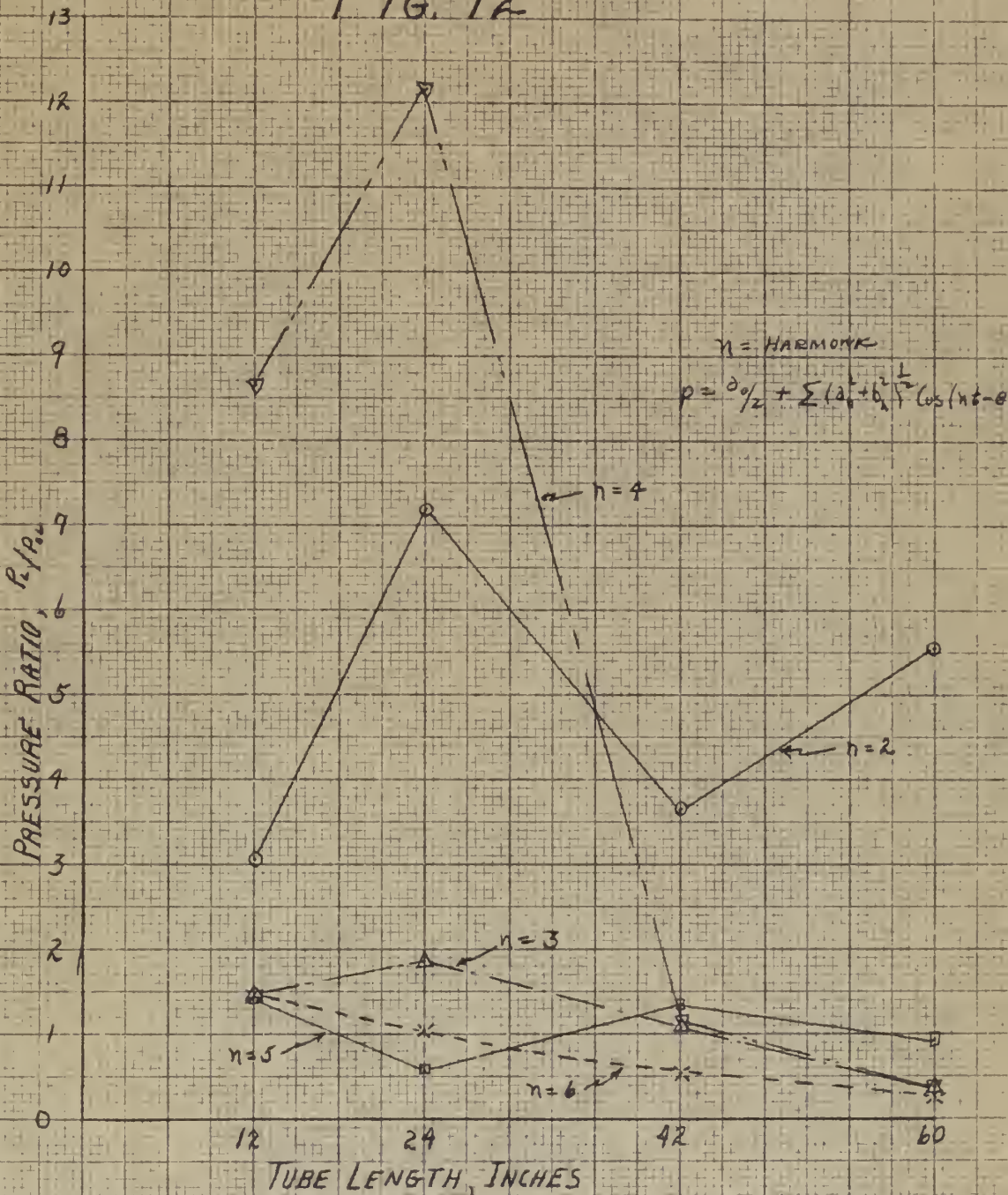FOR ZERO TUBE LENGTH AT 30 CYCLES PER SECOND

FIG. 11b

PHASE LAG OF FUNDAMENTAL HARMONIC COMPONENT
OF SQUARE PRESSURE WAVE AT 30 CYCLES PER
SECOND FOR FINITE TUBE LENGTHS

FIG. 12

$n = HARMONIC$

$p = \frac{a_0}{2} + \sum (a_n^2 + b_n^2)^{\frac{1}{2}} \cos(nt - \theta$

n = 4

n = 2

n = 3

n = 5

n = 6

PRESSURE RATIO, P_L/P_{zL}

TUBE LENGTH, INCHES

RATIOS OF PRESSURE AMPLITUDE FOR FINITE TUBE
LENGTH TO PRESSURE AMPLITUDE FOR ZERO TUBE LENGTH
FOR SECOND THROUGH SIXTH RESULTANT HARMONICS OF A
GENERATED SQUARE PRESSURE WAVE AT 30 CPS. STATIC
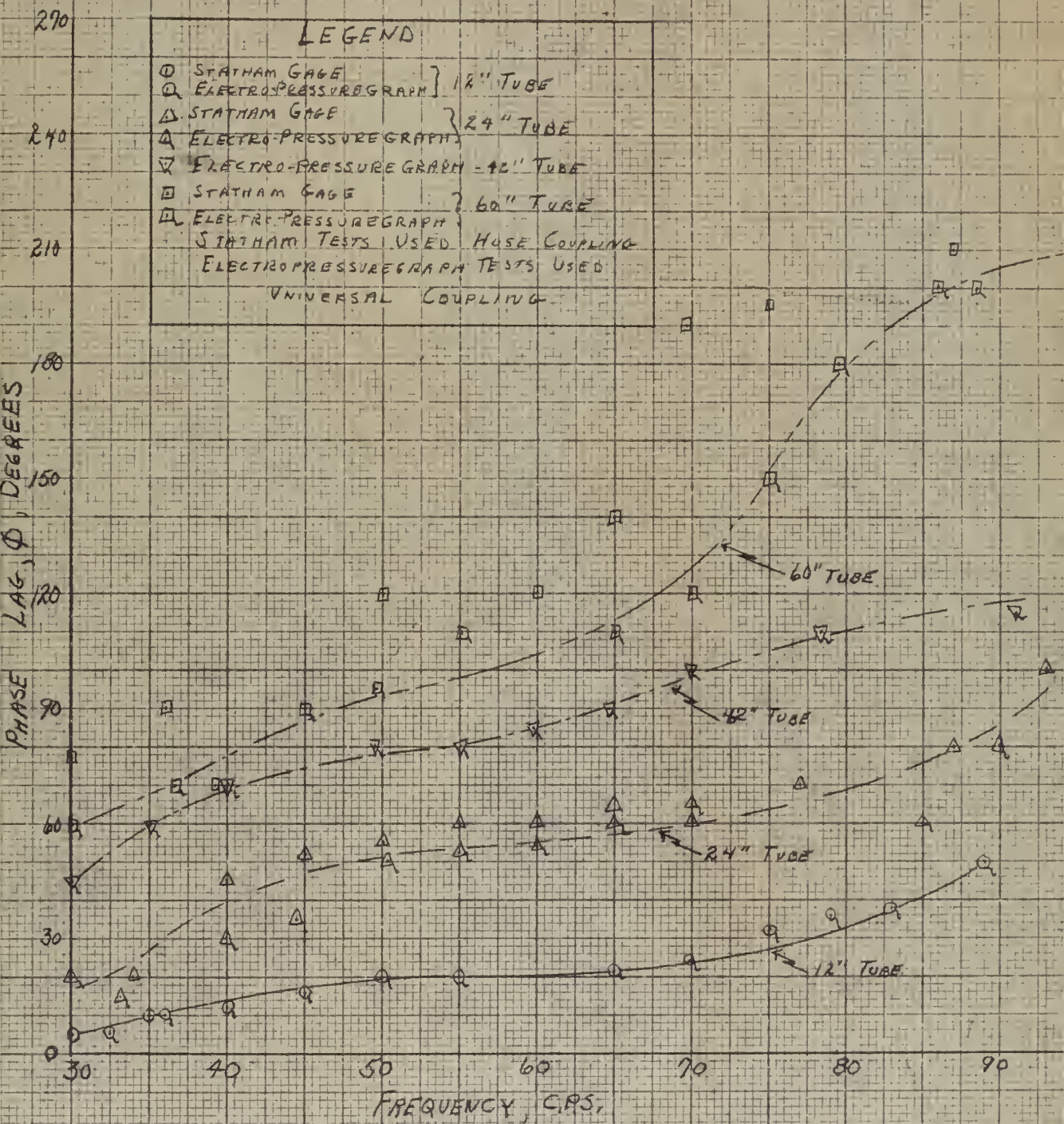PRESSURE SUPPLIED TO WAVE GENERATOR = 5.70 in. Hg.

LEGEND

⊕ STATHAM GAGE ⎤
⦺ ELECTRO-PRESSUREGRAPH ⎦ 12" TUBE

△ STATHAM GAGE ⎤
▲ ELECTRO-PRESSUREGRAPH ⎦ 24" TUBE

▽ ELECTRO-PRESSURE GRAPH – 42" TUBE

☐ STATHAM GAGE ⎤
▣ ELECTRO PRESSUREGRAPH ⎦ 60" TUBE

STATHAM TESTS USED HOSE COUPLING
ELECTROPRESSUREGRAPH TESTS USED
UNIVERSAL COUPLING

60" TUBE

42" TUBE

24" TUBE

12" TUBE

FREQUENCY, C.P.S.

FIG 13

PHASE LAG FOR FINITE TUBE LENGTHS AT VARIOUS
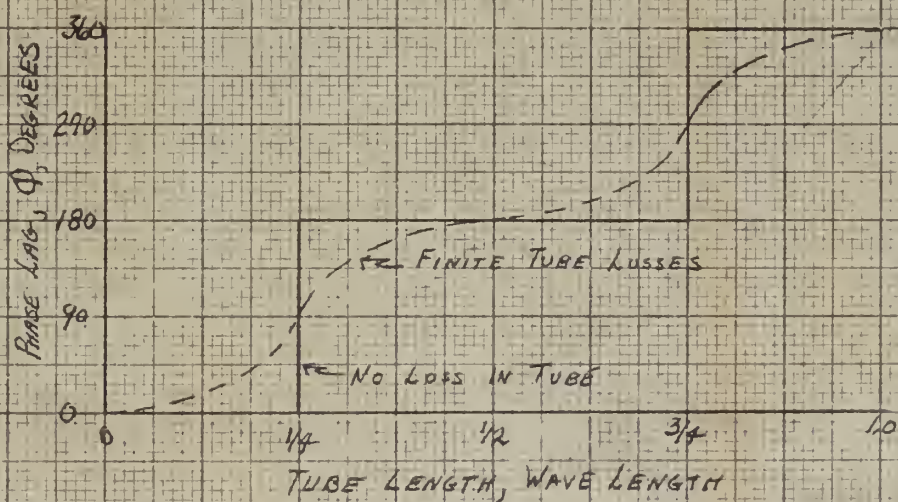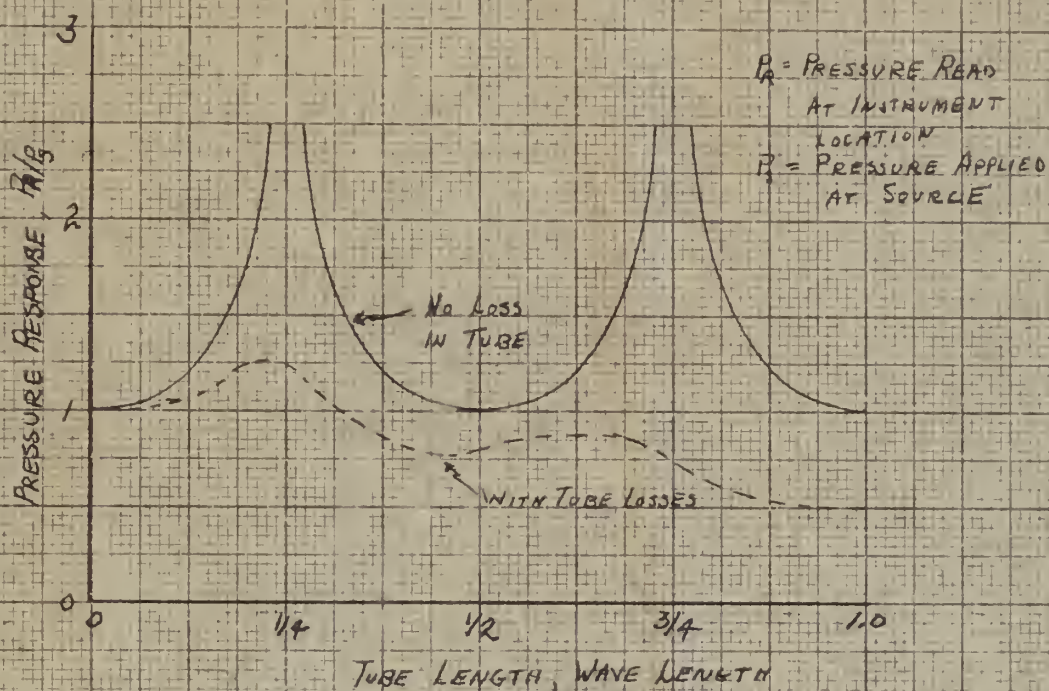FREQUENCIES AS DETERMINED DIRECTLY FROM
OSCILLOGRAMS OF A SQUARE PRESSURE WAVE

FIG 14

PRESSURE RESPONSE AND PHASE LAG FOR
VARIOUS TUBE LENGTHS FOR SINUSOIDAL
OSCILLATING PRESSURES
WITH NEGLIGIBLE INSTRUMENT
VOLUME AS OBTAINED FROM
A.C. THEORY ANALOGY (R-LC CIRCUIT)
(FROM NACA TN 1819, REFERENCE 3)